

MACROECONOMIC AND FISCAL POLICY IMPLICATIONS OF HOUSEHOLD

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## ABSTRACT

### MACROECONOMIC AND FISCAL POLICY IMPLICATIONS OF HOUSEHOLD LABOR SUPPLY

Chunzan Wu

Dirk Krueger

This dissertation uses dynamic macroeconomic models with household heterogeneity to study the implications of household labor supply on household consumption dynamics and fiscal policy. Chapter 1 (joint work with Dirk Krueger) studies how the endogenous household labor supply channel affects the ability of households to smooth consumption against exogenous wage shocks. We show that a calibrated life-cycle two-earner household model with endogenous labor supply can rationalize the extent of consumption insurance against wage shocks estimated empirically by Blundell, Pistaferri, and Saporta-Eksten (2014) (BPS hereafter) in the U.S. data. With additive separable preferences, only 41% of male and 28% of female permanent wage shocks in the model pass through to household consumption. Most notably, the majority of the consumption insurance against permanent male wage shocks is provided through the endogenous labor supply response of the female earner. We also evaluate, using model-simulated data, the performance of the empirical approach of BPS on consumption responses to wage shocks and find only moderate biases. Chapter 2 studies the implications of changes in economic fundamentals such as increased female labor productivity, skill-biased technological change and aging population on the changes

of the U.S. income tax code since the late 1970s. I first study these changes in economic fundamentals using an overlapping generations incomplete-markets life-cycle model with heterogeneous households. The model features both endogenous human capital accumulation and household labor supply and is calibrated to the U.S. economy in the 1970s and 2010s. Then I use this economic model to examine the income tax changes in a Ramsey optimal tax policy framework. I find that: (1) changes in economic fundamentals alone induce a less progressive optimal income tax and can account for 40% of the reduction in progressivity we observe; and (2) the change in Pareto weights required to explain the remaining part of tax policy change favors high-income households and also implies less valued government services. Finally, using a stylized political economy model, I discuss potential explanations for this change of Pareto weights such as the lower cost of conveying information to swing voters and the rising inequality of voter turnout among different socioeconomic groups.

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# Overview

This dissertation uses dynamic macroeconomic models with household heterogeneity to study the implications of household labor supply on household consumption dynamics and fiscal policy. It belongs to the broader research agenda of quantitative macroeconomics with heterogeneous agents, which stresses the incompleteness of financial markets and consequently the macroeconomic implications of idiosyncratic risks associated with individual economic activities. With incomplete financial markets and idiosyncratic risks, the cross-sectional distribution of households becomes an important state variable of the economy. Hence the typical approach in this direction of research is to first develop quantitative models capable of matching the empirical cross-sectional distributions of economic variables such as household consumption and income and then use the models as laboratories to address macroeconomic and policy questions. This is also the approach adopted in the studies of this dissertation.

The focus of this dissertation on household labor supply is motivated by the rising economic role of females within the U.S. households since the 1970s. Female labor supply in the U.S. has increased significantly relative to male labor supply through both the intensive and extensive margins since the 1970s. The wage gap between females and males has also declined during the same time period. As a result, female labor income has increased from about one third to two thirds of male labor income. Previous economic models usually assume one earner within each household, and these changes related to female labor supply

naturally make it a pressing task for economists to develop new models featured with two-earner households. As shown in the studies of this dissertation, the existence of a female earner within households and the differences between the male and the female earners such as labor supply elasticity will be critical in the discussions of many economic issues.

Chapter 1 of this dissertation (joint work with Dirk Krueger) studies the implications of household labor supply on household consumption dynamics. We show that a calibrated life-cycle two-earner household model with endogenous labor supply can rationalize the extent of consumption insurance against wage shocks estimated empirically by Blundell, Pistaferri, and Saporta-Eksten (2014) (BPS hereafter) in the U.S. data. With additive separable preferences, the model can account for about 94% and 91% of consumption insurance against male and female permanent wage shocks in the data, and only 41% of male and 28% of female permanent wage shocks in the model pass through to household consumption. With non-separable preferences, more consumption insurance is generated, and the pass-through rates are 27% and 18%, respectively. Most notably, the majority of the consumption insurance against permanent male wage shocks is provided through the endogenous labor supply response of the female earner. We also evaluate, using model-simulated data, whether the empirical approach of BPS delivers unbiased consumption responses to wage shocks. We find that the method overestimates the amount of consumption insurance against male permanent wage shocks, and underestimates that against female permanent wage shocks, but only moderately so. We find larger biases for the outside insurance coefficient which BPS use to capture the insurance provided through channels outside their model. We document that the magnitudes of the biases are not sensitive to the existence of tight borrowing constraints or the presence of an extensive female labor supply margin given their implementation method.

Chapter 2 of this dissertation studies the fiscal policy implications of household labor supply. Since the 1970s, income inequality in the U.S. has increased sharply. During the

same time span, the U.S. federal income tax has become *less* progressive. Why? I examine this question in a Ramsey optimal tax policy framework. Within this framework, the tax policy is determined by: (1) a set of Pareto weights representing the government's preference over different households; and (2) household lifetime utilities summarizing the effects of economic fundamentals. I first study the changes in economic fundamentals using an overlapping generations incomplete-markets life-cycle model with heterogeneous households. The model features both endogenous human capital accumulation and household labor supply and is calibrated to the U.S. economy in the 1970s and 2010s. Then I use this economic model to determine whether the change in income tax is the result of an optimal policy response to changing economic fundamentals or the consequence of a change in Pareto weights. I interpret the latter as changes in the political influences of various income groups. I find that: (1) changes in economic fundamentals alone induce a less progressive optimal income tax and can account for 40% of the reduction in progressivity we observe; and (2) the change in Pareto weights required to explain the remaining part of tax policy change favors high-income households and also implies less valued government services. Finally, using a stylized political economy model, I discuss potential explanations for this change in Pareto weights such as the lower cost of conveying information to swing voters and the rising inequality of voter turnout among different socioeconomic groups.

## References

Blundell, Richard, Luigi Pistaferri, and Itay Saporta-Eksten (2014). *Consumption Inequality and Family Labor Supply*. Working Paper.

# Chapter 1

## How Much Consumption Insurance in Bewley Models with Endogenous Family Labor Supply?<sup>1</sup>

Chunzan Wu and Dirk Krueger

### 1.1 Introduction

How does household consumption respond to shocks to wages of the primary earner? The baseline version of the permanent income hypothesis in which a household has only one bread winner and labor supply of the primary earner is exogenous provides a sharp answer: household consumption responds to permanent wage shocks one for one, and essentially not at all to purely transitory shocks. In a sequence of important papers, Blundell, Pistaferri, and Preston (2008) and Kaplan and Violante (2010) propose to measure the magnitude of the household consumption response to earnings shocks with various persistence

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<sup>1</sup>We thank Greg Kaplan and Luigi Pistaferri for useful conversations at an early stage of this project.

by consumption insurance coefficients. This measure is defined as the fraction of the variance of a given shock to log-earnings that does *not* translate into a corresponding change in log-consumption.<sup>2</sup> That is, if the consumption insurance coefficient for a given earnings shock is 1, household consumption growth is completely insulated from the earnings shock, and if it is zero, the earnings shock translates one for one into consumption. Blundell, Pistaferri, and Preston (2008) empirically estimate these consumption responses to transitory and permanent earnings shocks on U.S. data and find close to perfect insurance against purely transitory shocks (except for poor households) as well as substantial insurance against permanent shocks, with a consumption insurance coefficient of 35%.<sup>3</sup> Kaplan and Violante (2010) evaluate whether a calibrated incomplete-markets life cycle model (SIM henceforth) in which a single-earner household faces transitory and permanent earnings shocks is consistent with the empirical evidence provided by Blundell, Pistaferri, and Preston (2008). They find that in the model households are close to fully insured against transitory earnings shocks, but that there is too little consumption insurance against permanent shocks: the model-implied consumption insurance coefficient ranges between 7% and 22%, depending on how tight borrowing constraints are assumed to be, and thus is significantly smaller than the estimates of Blundell, Pistaferri, and Preston (2008).

In both Blundell, Pistaferri, and Preston (2008) and Kaplan and Violante (2010), as in much of the empirical and model-based consumption literature household earnings are treated as exogenous, and the key mechanism through which consumption insurance is materialized is saving (and borrowing, if permitted) at a state uncontingent interest rate.

<sup>2</sup>Formally, denoting by  $c_{it}$  the log of consumption of household  $i$  at time or age  $t$ , the consumption insurance coefficient for earnings shock  $x_{it}^n$  of type (persistence)  $n$  is defined as

$$\phi_t^n = 1 - \frac{Cov_i(\Delta c_{it}, x_{it}^n)}{Var_i(x_{it}^n)} \quad (1.1.1)$$

where  $Cov_i$  and  $Var_i$  are taken with respect to the cross-section of household observations indexed by  $i$  observed at time (or age)  $t$ .

<sup>3</sup>Also see Santaella-Llopi and Zheng (2014) for a recent application of the same method to Chinese data.

By construction this literature is therefore silent about the underlying shocks behind the earnings fluctuations as well as alternative mechanisms by which households can and do respond to these underlying shocks. The current paper aims at contributing to the literature by modeling the fundamental sources of consumption risk in an otherwise standard SIM with idiosyncratic shocks (of various persistence) to wages of the male and female earner of a two-member household. As in Kaplan and Violante (2010) our primary objective is to quantify the extent to which wage shocks translate into consumption movements, but with specific focus on the importance of *alternative mechanisms* (adjustment of labor hours of both members of the household and participation of the female earner, as well as precautionary savings) by which consumption insurance occurs in the model.

By doing so we can assess whether our model can match well the empirically estimated labor supply and consumption responses to transitory and permanent wage shocks in the recent paper by Blundell, Pistaferri, and Saporta-Eksten (2014), henceforth BPS. In this work, which can be seen as the natural extension of Blundell, Pistaferri, and Preston (2008) to endogenous household labor supply the authors empirically estimate the transmission coefficients from transitory and permanent wage shocks to labor earnings and consumption<sup>4</sup> in two-earner (male and female) households. We treat their estimates as the empirical benchmark against which the transmission coefficients estimated from model-simulated data should be compared, very much in the same spirit Kaplan and Violante (2010) used the empirical estimates of Blundell, Pistaferri, and Preston (2008).

Our findings suggest that the model fits the data reasonably well. In other words, a Bewley type model with two-earner households and endogenous labor supply can explain most of the responses of household consumption and labor income with respect to wage shocks in the data. This result is prominent given the simplicity of the model, and pro-

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<sup>4</sup>For consumption, their transmission coefficients have exactly the same interpretation as the consumption insurance coefficients discussed above, but now understood as consumption insurance against wage rather than earnings shocks. With exogenous labor supply and single earner households the two coincide exactly.



vides a solid justification for using such model in economic studies of pertinent issues. In the calibrated model with an additive separable preference between consumption and labor supply, about 41% of male permanent wage shocks and 28% of female permanent wage shocks pass through to household consumption. The empirical estimates of them from BPS are 37% and 21%, respectively. Hence the model can already account for about 94% and 91% of consumption insurance against male and female permanent wage shocks observed in the data. The insurance against transitory wage shocks is almost perfect in the model with close-to-zero pass-through rates for both male and female shocks, while the BPS counterparts are slightly negative but not statistically different from zero given their standard errors.<sup>5</sup> The model also reproduces well the labor income responses with respect to wage shocks in the data. Both the model and the BPS results indicate a rising consumption insurance against permanent wage shocks over the life cycle, which is caused by the improved asset positions of old households. A decomposition of consumption insurance against male permanent wage shocks in the model shows that the female earner provides most of the insurance and her contribution is almost constant over the life cycle, while the male earner provides increasingly negative insurance over the life cycle. BPS suggest that there are some evidences of complementarity between consumption and leisure, and between leisures of the spouses. Hence we have also considered a non-separable preference between consumption and labor supply with the constant elasticity of substitution (CES) functional form calibrated to match such complementarity. The non-separability between consumption and labor supply in preference increases the amount of consumption insurance against permanent wage shocks, and the pass-through rates of male and female permanent wage shocks are now 27% and 18%, which are slightly lower than the BPS estimates. The non-separability also helps the model to match the negative pass-through rates

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<sup>5</sup>Throughout this paper, we choose 1% as the common significance level for statistical tests and statements.

of transitory wage shocks to consumption in the BPS results. Most of the conclusions for the additive separable preference case remain valid for the non-separable preference case.

In order to make sure that this comparison between our model results and the empirical evidence is not contaminated by the possible biases of the estimation method Blundell, Pistaferri, and Saporta-Eksten (2014) employ, we also evaluate their empirical approach using model-simulated data. Our results suggest that the estimation method BPS employ performs reasonably well on simulated data if implemented properly, and hence the conclusions previously cited about the comparisons between the model-based and the empirical BPS results are not significantly affected by the biases of the estimation method. The BPS method tends to overestimate the consumption insurance against transitory wage shocks and male permanent wage shocks, but to underestimate that against female permanent wage shocks. However, the magnitudes of these biases are small, between 0.1% and 3% in absolute value. We find the estimation of the consumption own Frisch elasticity is sensitive to the moment conditions used which could translate into larger biases on consumption insurance, but it can be improved significantly by using multiple moment conditions and iterated GMM method with updated weighting matrix. We find the iterated GMM method performs better than the one-step GMM method used by BPS, especially when the preference is non-separable. The estimation of the outside insurance, if allowed, is unstable and significant biased, and hence should be avoided. The derivation of the BPS estimators from a theoretical model with endogenous labor supply and incomplete asset markets requires interior solutions of the household's maximization problem, which is not always assured when borrowing constraints are potentially binding or an operative extensive margin of female labor supply exists. However, our study shows that the performance of the BPS estimators is not affected much by the violations of these assumptions. This is mainly due to the restriction of BPS to households aged between 30 and 57, and most of these households are no longer bound by the tight borrowing constraints. The extensive margin of female

labor supply induces slightly larger biases on female related estimates, but the impacts are limited because the female nonparticipation rate is moderate in the BPS data set. Hence it is not a significant source of bias either.

In terms of the related literature, clearly the contributions by Blundell, Pistaferri, and Preston (2008), Kaplan and Violante (2010) and Blundell, Pistaferri, and Saporta-Eksten (2014) are most directly relevant for our paper. More broadly, our work is related to the literature that has studied heterogeneous household models with idiosyncratic risks, as in Huggett (1993) and Aiyagari (1994).

The structural life-cycle model we employ is most closely related to the models analyzed by Heathcote, Storesletten, and Violante (2010) and Holter, Krueger, and Stepanchuk (2014). However, their applications mainly focus on inequality and tax policy rather than the consumption insurance question we address here, and they do not consider the possibility of non-separable preferences. The paper is also related to the literature on within-household risk-sharing and the role female labor supply plays in this context. For example, Attanasio, Low, and Sánchez-Marcos (2005) study the importance of female labor supply as an insurance mechanism against idiosyncratic income risk within the family, but in their model the labor supply decision is discrete and the intensive margin of labor supply is absent. Ortigueira and Siassi (2013) investigate the impact of within-household risk-sharing on household labor supply and savings. However, only idiosyncratic unemployment risk is considered. To our knowledge, no previous work has examined the issue of consumption insurance against wage shocks in a structural model of two-earner households with endogenous labor supply of both household members.<sup>6</sup>

The rest of this paper proceeds as the following: Section 2.2 describes the model, preference choices and calibration strategy; Section 1.3 introduces the transmission coefficients

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<sup>6</sup>Perhaps closest in this regard is Heathcote, Storesletten, and Violante (2014) who study consumption insurance against wage shocks in an economy of single households with endogenous labor supply and within-group risk-sharing.

and offers a brief review of the BPS method; Section 2.4 reports and discusses the main results; Section 1.5 evaluates the performance of the BPS method; Section 2.6 concludes.

## 1.2 Model

### 1.2.1 The Environment and Household's Problem

We study a partial equilibrium life cycle model with idiosyncratic wage risk. The model economy is populated with a continuum of two-earner households at different ages. The measure of households at each age is set to be 1. Each household lives for  $T$  periods from age  $t = 1$  to  $T$ . They work in the first  $R$  periods of their life and then retire from age  $R + 1$ . Each household has two members: the male and the female. In the following, we use the notation  $X_{j,t}$  to denote the variable  $X$  of earner  $j$  at age  $t$ , with  $j = 1$  and  $2$  corresponding to the male and female members.

At each period, households receive utility from joint household consumption,  $C_t$ . A working household's utility is also affected by the levels of their labor supply,  $H_{1,t}$  and  $H_{2,t}$ . Hence the instantaneous utility function is assumed to be  $u(C_t, H_{1,t}, H_{2,t})$  for a working household and  $u^R(C_t)$  for a retired household. The two members of each household are assumed to make joint decisions on consumption and labor supply. Both members of a working households can work and receive wages determined by their own labor productivity. The wages of two members in each household,  $W_{j,t}$ , are stochastic, and the log of them can be represented by

$$\log W_{j,t} = g_{j,t} + F_{j,t} + u_{j,t}$$

$$F_{j,t} = F_{j,t-1} + v_{j,t}$$

$$\begin{bmatrix} v_{1,t} \\ v_{2,t} \end{bmatrix} \sim iid N \left( 0, \begin{bmatrix} \sigma_{v_1}^2 & \sigma_{v_1,v_2} \\ \sigma_{v_1,v_2} & \sigma_{v_2}^2 \end{bmatrix} \right)$$

$$\begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix} \sim iid N \left( 0, \begin{bmatrix} \sigma_{u_1}^2 & \sigma_{u_1, u_2} \\ \sigma_{u_1, u_2} & \sigma_{u_2}^2 \end{bmatrix} \right)$$

where  $g_{j,t}$  is the life-cycle trend of  $\log W_{j,t}$ ;  $F_{j,t}$  is a permanent component and  $u_{j,t}$  is a transitory component. Hence the innovation to the permanent component,  $v_{j,t}$ , is the permanent shock to earner  $j$ 's wage, and the innovation to the transitory component,  $\Delta u_{j,t} = u_{j,t} - u_{j,t-1}$ , is the transitory shock. Both  $v_{j,t}$  and  $u_{j,t}$  can be correlated between the two members of each household but are assumed to be independent with each other and over time. After retirement, there are no wage shocks and the levels of labor supply are forced to be zero. A retired household receives a fixed amount of social security benefits,  $b$ , in each period. We abstract from the progressive income tax in the model because BPS find that adding this feature does not significantly change the results.

Households can trade a risk free bond with interest rate  $r$ , but suffer from age-dependent borrowing constraints: the asset position of a household at age  $t$ ,  $A_t$ , must be above a predetermined level  $\underline{A}_t$ , i.e.,  $A_t \geq \underline{A}_t, \forall t$ . There are no state-contingent bonds available, so the financial markets are incomplete.

A working household's problem is then in recursive form

$$V(A, F_1, F_2, u_1, u_2, t) = \max_{C, A', H_1, H_2} u(C, H_1, H_2) + \frac{1}{1 + \delta} \sum_{(F'_1, F'_2)} \pi(F'_1, F'_2 | F_1, F_2) \sum_{(u'_1, u'_2)} \pi(u'_1, u'_2) V(A', F'_1, F'_2, u'_1, u'_2, t + 1)$$

s.t.

$$C + A' = W_{1,t}H_1 + W_{2,t}H_2 + (1 + r)A$$

$$C, H_1, H_2 \geq 0, A' \geq \underline{A}_{t+1}$$

where  $\frac{1}{1+\delta}$  is the time discount factor and  $\pi(\cdot|\cdot)$  is the transition probability function for

shocks.

And a retired household's problem is

$$V^R(A, t) = \max_{C, A'} u^R(C) + \frac{1}{1 + \delta} V^R(A', t + 1)$$

s.t.

$$C + A' = b + (1 + r)A$$

$$C \geq 0, A' \geq \underline{A}_{t+1}$$

where  $u^R(C)$  is the utility function for retired households.

## 1.2.2 The Choice of the Utility Function

There is no consensus so far about the functional form of the instantaneous utility function when the disutility of labor supply is added. A widely used one in the life-cycle model literature is the additive separable preference:

$$u(C, H_1, H_2) = \frac{C^{1-\sigma}}{1-\sigma} - \psi_1 \frac{H_1^{1+\eta_1^{-1}}}{1+\eta_1^{-1}} - \psi_2 \frac{H_2^{1+\eta_2^{-1}}}{1+\eta_2^{-1}}.$$

The advantages of using this preference are mainly the computational simplicity and the clear interpretation of preference parameters. With the additive separable preference between consumption and labor supply, the marginal utility of consumption is independent from the levels of labor supply. Hence the Euler equation of consumption remains the same as that in a model with the constant relative risk aversion (CRRA) utility function but no labor supply. Also, the optimal levels of labor supply can be easily pinned down by the intratemporal optimality conditions, and are simple functions of the optimal consumption level. Hence models with this type of preference are easily solved numerically. The pa-

parameter  $\sigma$  governs the intertemporal elasticity of substitution (IES) for consumption, and the reciprocal of it is the Frisch elasticity of consumption with respect to its own price. The parameter  $\eta_1$  and  $\eta_2$  are Frisch elasticities of male and female labor supply with respect to their own wages. However, the disadvantage is that this preference does not allow for a possible interaction between labor supply and consumption in preference. Indeed, BPS argue that the non-separability between consumption and labor supply is useful for a better fit of the data. Therefore we also consider a non-separable preference with the constant elasticity of substitution (CES) functional form:

$$u(C, H_1, H_2) = \frac{\{\alpha C^\gamma + (1 - \alpha)[\xi H_1^\theta + (1 - \xi)H_2^\theta]^{-\frac{\gamma}{\theta}}\}^{\frac{1-\sigma}{\gamma}} - 1}{1 - \sigma} \quad (1.2.1)$$

where  $\gamma$  governs the substitution pattern between consumption and labor supply, and  $\theta$  governs the substitution pattern between the male and female labor supply. The main advantage of this utility function is that it is flexible enough to accommodate different substitution patterns while keeps the computation burden manageable. The optimal levels of labor supply can still be represented explicitly as functions of the optimal consumption level. However, the disadvantage is that the simple mappings between the preference parameters and Frisch elasticities are lost in the sense that the Frisch elasticities are no longer deep parameters and depend on the allocations chosen by households.

Given the fact that a significant proportion of the female do not participate in the labor market, an operative extensive margin of female labor supply is included in the model by adding a fixed utility cost to the instantaneous utility when the female labor supply is strictly positive. That is, we add a term  $-I(H_2 > 0)f$  to the instantaneous utility where  $I(\cdot)$  is the indicator function and  $f$  is the fixed cost.

The instantaneous utility function for a retired household is less controversial because labor supply is irrelevant now. We choose the CRRA utility function for retired households:

$$u^R(C) = \Psi \frac{C^{1-\sigma} - 1}{1 - \sigma}$$

where  $\Psi$  is a constant which governs the scale of marginal utility of consumption after retirement.<sup>7</sup>

### 1.2.3 Calibration Strategy

In BPS, the data used are from 1999-2009 Panel Study of Income and Dynamics (PSID). PSID collected data from two groups of households: one group was representative of the US population as a whole; the other group was the low income households, i.e., Survey of Economic Opportunity (SEO) sample. The estimation was done using data on the non-SEO households with participating and married male household heads aged between 30 and 57. Because we want to compare the results of our model with the BPS estimates, the model is calibrated to match the statistics from this group of households if possible. This calibration strategy gives the best hope of the model to fit the BPS estimates. Hence if we found any significant differences between our models and the BPS empirical results, they were more likely to be caused by the structure of the model rather than inappropriate parameter values. More details about the calibration results are available in Appendix 1.B.

**Demographic and Initial Wealth.** Households are assumed to be born at age 21, retire at age 65 and die at age 80. So age 1 in the model corresponds to age 21 in reality, and  $T$  and  $R$  are 60 and 45, respectively. The initial household asset level is set to be zero.

The median age at first marriage in the US between 2000 and 2010 is about 27.5 for male and 26 for female according to the US census data. In this sense, the starting age of 21 for households in the model is a little earlier. There are mainly two reasons why we chose

<sup>7</sup>For the additive separable preference,  $\Psi$  is simply set to be 1. For the non-separable preference,  $\Psi$  is set such that the age profile of consumption is smooth at retirement. While consumption typically falls upon retirement, previous literature on retirement consumption puzzle shows that it is mainly due to work related consumption which is not modeled here. See Hurst (2008) for more details about this literature.



this. First, while the age at the actual marriage is older, the starting age of cohabitation could be much younger, when the within-household risk-sharing actually starts. Second, by the time of official marriage, the couples are likely to have already accumulated some assets and have permanent wage components determined by previously realized sequences of shocks. It is difficult to identify the permanent components of wages at individual level, so it is not feasible to get the exact empirical joint distribution of the permanent components of wages and the asset level as the starting states of simulated households. As a compromise, we assume that the households start from an earlier age when the asset level is zero. When calculating the relevant results, only the simulated data of households with age 30 to 57 are used, which is the same age group used by BPS, so the choice of the initial conditions should not be too critical.

**Wage Process.** The life cycle profile of wage is taken from Rupert and Zanella (2012). They estimate this wage profile from PSID 1967-2008. The original wage profile starts from age 23 and only has values biennially after age 52, so we interpolate it to an annual profile and extend the age range to 21-65. Because Rupert and Zanella (2012) report only the pooling wage profile of the male and female, and the estimation of female wage profile often suffers from the selection bias of female participation, we assume in the model that the wage profiles of male and female earners have the same life cycle shape but different levels.<sup>8</sup> The average of the male wage profile is normalized to be 1. In the BPS data set, the female average wage is about 68.5% of the male average wage, so the level of the female wage profile is calibrated to match this ratio.

The covariance matrices of transitory and permanent wage shocks are taken directly from the BPS estimates. Both the permanent and transitory wage shocks are assumed to be i.i.d. across time but potentially positively correlated between the two earners of each

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<sup>8</sup>Because the wage trends are perfectly foreseeable by households in the model, the behavioral response of households with respect to unexpected shocks, which is the main focus of this paper, should not be affected much by the shape of the wage trends.

household.

$$\begin{bmatrix} \sigma_{u_1}^2 & \sigma_{u_1,u_2} \\ \sigma_{u_1,u_2} & \sigma_{u_2}^2 \end{bmatrix} = \begin{bmatrix} 0.0297 & 0.0054 \\ 0.0054 & 0.0125 \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{v_1}^2 & \sigma_{v_1,v_2} \\ \sigma_{v_1,v_2} & \sigma_{v_2}^2 \end{bmatrix} = \begin{bmatrix} 0.0294 & 0.0028 \\ 0.0028 & 0.0391 \end{bmatrix}$$

**Borrowing Limit.** It is unclear what should be the proper borrowing limits in reality, so we consider two extreme cases: zero borrowing constraints (ZBC) and not-binding borrowing constraints (NBBC). It is reasonable to believe that the true state of the world is between these two cases. The ZBC case is simply to set the borrowing limits to be zero at each age. The NBBC case is much trickier. One convention is to use the natural borrowing limits as in Kaplan and Violante (2010) for very relaxed borrowing constraints. The idea is that agents should be able to pay back their debts with probability one if they hit the constraints, and the natural borrowing constraints are the lower bounds of these constraints. However, with unbounded endogenous labor supply, households can pay back any amount of debts by working long enough hours. As a result, the idea of the natural borrowing limits is no longer appropriate. To fix this problem, we use a different set of not-binding borrowing constraints. In particular, we first solve the model with negative constant borrowing constraints, and check by simulation that no households are ever borrowing constrained. Then we trace down the lowest asset positions reached by different age groups, and use those as a sequence of age-dependent borrowing constraints to solve the model again. This procedure can be repeated several times to improve the accuracy.

**Discount Factor and Interest Rate.** The interest rate is taken from the BPS which is 2% per year. The discount factor is going to affect households' incentive to save which is an important channel to insure against wage shocks. So we calibrate the discount rate  $\delta$

such that the aggregate asset-income ratio for age 30 to 57 households in the model is about 2.83 which is the counterpart value in the BPS data set.

**Social Security Benefits.** In reality, the US social security benefits are non-linear segmented functions of the Average Indexed Monthly Earnings (AIME), and there are additional rules to the spouse's benefits. A full characterization of the US social security rules is costly in terms of computation and the complexity of the model. Hence we assume that the retirement benefit per household,  $b$ , in the model is the same for every household, and calibrate the value of it to match the ratio of average social security benefits per household to average household income. In 2012, the monthly average social security payment to a retired worker is 1262 dollars (Social Security Administration report 2013), and the household annual income mean is 71274 dollars (Current Population Survey 2013). Hence the ratio of the retirement benefit per household to average household income in the model is calibrated to  $\frac{1262 \times 2 \times 12}{71274} = 0.4250$ .

**Fixed Utility Cost of Positive Female Labor Supply.** In reality, both the male and female have non-trivial non-participation rates in the labor market. However, because the BPS results are based on a sample of households with a working male member, we do not include the extensive margin of male labor supply in the model. To take into account the female extensive margin in the model, the fixed utility cost of positive female labor supply  $f$  is chosen such that the average female non-participation rate of age 30 to 57 households is the same as that from the BPS data set which is 21%.

**Additive Separable Preference.** With the additive separable preference,  $\frac{1}{\sigma}$ ,  $\eta_1$ , and  $\eta_2$  are just the consumption, male and female labor supply Frisch elasticities with respect to their own prices, i.e.,  $\eta_{c,p}$ ,  $\eta_{h_1,w_1}$  and  $\eta_{h_2,w_2}$ .<sup>9</sup> So their values are directly taken from the BPS

<sup>9</sup>Following the original BPS paper, Frisch elasticities are denoted by  $\eta$  with corresponding subscripts. The meanings of subscripts are  $c$  for consumption,  $h_j$  for earner  $j$ 's labor supply,  $p$  for the price of consumption and  $w_j$  for earner  $j$ 's wage. For example,  $\eta_{c,p}$  corresponds to the Frisch elasticity of consumption with respect to its own price.

estimates with the separability assumption. In particular,  $\sigma = 1/0.613$ ,  $\eta_1 = 0.490$ , and  $\eta_2 = 0.849$ . The values of  $\psi_1$  and  $\psi_2$  are calibrated to match the average male and female labor supply implied by the data. For the male, the mean of labor supply is normalized to be 1, which means 1 unit of labor supply in the model corresponds to the average hours worked by the male in the BPS data set, i.e., 2202 hours per year. The average hours worked by the female conditional on working in BPS data set is about 1698 hours, hence the target value of the average female labor supply conditional on working is 0.771 in the model.

**Non-separable Preference.** With the non-separable preference, Frisch elasticities are no longer deep parameters. Given the functional form in this paper, we are able to derive the Frisch elasticities as functions of preference parameters and allocations. The formulas are shown in Appendix 1.C. The preference parameters are then calibrated jointly to match the Frisch elasticities estimated by BPS without the separability assumption and the average hours worked by the male and female in the BPS data set. The parameters  $\gamma$ ,  $\theta$  and  $\sigma$  mainly affect the Frisch elasticities, while the parameters  $\alpha$  and  $\xi$  mainly affect the labor supply. In particular, for a given set of parameters, the model is first solved and a panel of household data are simulated. Then based on the simulated data and the formulas for Frisch elasticities, the sample averages of Frisch elasticities and labor supply are calculated and compared with the calibration targets. One issue is that given the choice of functional form for preference, there are not enough degrees of freedom to match all the Frisch elasticities perfectly.<sup>10</sup> We put more weights on those elasticities which capture the interaction between labor supply and consumption in utility<sup>11</sup> because that is the main difference between non-separable and additive separable preferences. The Frisch elasticities from BPS and the calibration results are available in Appendix 1.B.

<sup>10</sup>Actually, such utility function may not even exist due to the restrictions between Frisch elasticities imposed by their definitions.

<sup>11</sup>In particular,  $\eta_{c,w_1}$ ,  $\eta_{c,w_2}$ ,  $\eta_{h_1,w_2}$ ,  $\eta_{h_2,w_1}$  and  $\eta_{c,p}$ .

### 1.3 Transmission Coefficients and the BPS Method

In this section, we review the concept of the transmission coefficients and provide a brief description of the BPS method used to estimate these coefficients from PSID data. We also discuss possible biases and concerns associated with this method and how our exercises can help answer these questions. All the key formulas relevant to this paper and the details of how we implement the BPS method are provided in Appendix 1.D. For more information about this method, we refer the readers to the original BPS paper.

The focus of this paper is on how household consumption and labor income respond to unexpected wage shocks, the first order effects of which are captured by the transmission coefficients,  $\kappa$ 's, in Equation (1.3.1).

$$\begin{bmatrix} \Delta c_t \\ \Delta y_{1,t} \\ \Delta y_{2,t} \end{bmatrix} = \begin{bmatrix} \kappa_{c,u_1} & \kappa_{c,u_2} & \kappa_{c,v_1} & \kappa_{c,v_2} \\ \kappa_{y_1,u_1} & \kappa_{y_1,u_2} & \kappa_{y_1,v_1} & \kappa_{y_1,v_2} \\ \kappa_{y_2,u_1} & \kappa_{y_2,u_2} & \kappa_{y_2,v_1} & \kappa_{y_2,v_2} \end{bmatrix} \begin{bmatrix} \Delta u_{1,t} \\ \Delta u_{2,t} \\ v_{1,t} \\ v_{2,t} \end{bmatrix} \quad (1.3.1)$$

The left hand side variables of this equation include the consumption response,  $\Delta c_t$ , and the labor supply response,  $\Delta y_{j,t}$  to wage shocks of various persistence.<sup>12</sup> On the right hand side are the transitory wage shocks,  $\Delta u_{j,t}$ , and permanent wage shocks,  $v_{j,t}$ . The transmission coefficients measure directly how consumption and labor income respond to wage shocks of different persistence. For example,  $\kappa_{c,v_j}$  measures how consumption responds to earner  $j$ 's permanent wage shocks. A  $\kappa_{c,v_j}$  value of 0.4 means that 40% of earner  $j$ 's permanent wage shocks pass through to household consumption, and hence 60% of them are insured.

If transitory and permanent wage shocks are separately observable, equation (1.3.1),

<sup>12</sup>Note that all the changes here are measured in percentage term, i.e., first differences of logs. Also note that  $c_t$  and  $y_{j,t}$  are the residuals of log consumption and log labor income of earner  $j$  at age  $t$  after controlling the effects of observables.

and thus the insurance coefficients, could be estimated directly. However, in practice, only the sum of transitory and permanent wage shocks are observed directly in the data:

$$\Delta w_{j,t} = \Delta u_{j,t} + v_{j,t}.$$

Because  $\Delta w_{j,t}$  is only observed once for each individual at each time, it is technically impossible to recover the two types of shocks without additional information. One contribution of BPS is to provide an empirically applicable method to estimate these transmission coefficients. The authors show that if one log-linearizes the first order conditions (assuming interior solutions) and the intertemporal budget constraint of a two-earner household life cycle model very similar to the one described in Section 1.2.1, the authors are able to derive formulas for the transmission coefficients as functions of Frisch elasticities and smoothing parameters which can be calculated directly from the data. For example, the transmission coefficient  $\kappa_{c,v_1}$  can be calculated using

$$\kappa_{c,v_1} = \frac{(-\eta_{c,p} + \eta_{c,w_1} + \eta_{c,w_2})[\eta_{c,w_1} - (1 - \beta)(1 - \pi_t)(s_{1,t} + \overline{\eta_{h,w_1}})]}{\eta_{c,p} - \eta_{c,w_1} - \eta_{c,w_2} + (1 - \beta)(1 - \pi_t)(\overline{\eta_{h,p}} + \overline{\eta_{h,w_1}} + \overline{\eta_{h,w_2}})} + \eta_{c,w_1} \quad (1.3.2)$$

where the  $\eta$ 's and  $\overline{\eta}$ 's are Frisch elasticities and combinations of them.<sup>13</sup> As the authors argue, the entity  $\pi_t$  is well approximated by the share of asset in the total discounted wealth for a household at age  $t$ , that is:

$$\pi_t \approx \frac{Asset_t}{Asset_t + Human\ Wealth_t}$$

where human wealth is the expected value of the discounted future labor income stream of the household. Whereas  $s_{j,t}$  is approximately the share of earner  $j$ 's human wealth in the

<sup>13</sup>In particular,  $\overline{\eta_{h,p}} \equiv \sum_{j=1}^2 s_j \eta_{h_j,p}$ ,  $\overline{\eta_{h,w_1}} \equiv \sum_{j=1}^2 s_j \eta_{h_j,w_1}$  and  $\overline{\eta_{h,w_2}} \equiv \sum_{j=1}^2 s_j \eta_{h_j,w_2}$ .

total human wealth of the household, i.e.,

$$s_{j,t} \approx \frac{Human\ Wealth_{j,t}}{\sum_{i=1}^2 Human\ Wealth_{i,t}}$$

BPS add the parameter  $\beta$  to capture sources of insurance for the household that are not explicitly modeled (neither by them nor by us), such as the insurance provided by networks of relatives and friends as well as social insurance such as unemployment benefits and food stamps. The higher the values of  $\pi_t$  and  $\beta$  are, the less does consumption respond to wage shocks, and therefore the better these shocks are insured. The baseline results of BPS are estimated assuming  $\beta = 0$ .

To estimate the transmission coefficients, the BPS method includes four steps: (1) Estimate the variance-covariance matrices of the permanent and transitory shocks directly from wage data alone (with results documented in section 1.2.3). (2) Calculate the smoothing parameters  $\pi_t$  and  $s_{j,t}$  directly from the asset and labor income data. (3) Conditional on the results obtained from the first two steps, and using the empirical second order moments for  $\Delta c_t$  and  $\Delta y_{j,t}$ , a Generalized Method of Moment (GMM) is employed to jointly estimate all the Frisch elasticities  $\eta$ 's (and the external insurance coefficient  $\beta$  if assumed unknown). (4) Calculate the estimates of the transmission coefficients for each household using the formulas derived, and take the sample averages of them as the final results.

For the general case with non-separability between consumption and labor in the utility function no prior restrictions are imposed on the Frisch elasticities to be estimated. However, the assumption of separability in the utility function translates into the restrictions for the cross Frisch elasticities to be zero in the GMM estimation.

Several assumptions are imposed along the way in the BPS approach which delivers a transparent and operational methodology without imposing a specific functional form of the utility function. Violations of these assumptions of course may result in biases in the

estimation results. For example, in order to log-linearize the Euler equations requires interior solutions to the saving and labor supply decisions from the households' problem. This assumption does not hold if borrowing constraints of households are frequently binding or one of the household member decides not to participate in the labor market for a good number of households.<sup>14</sup>

As part of the contribution of the paper we evaluate, in section 1.5, the quality of the BPS methodology for environments with endogenous labor supply using *simulated* data from our life cycle model for which we know the true parameters, Frisch elasticities and thus partial insurance coefficients. But first we study, in the next section 2.4, and after briefly documenting basic life cycle profiles from the model, the model-implied wage-shock insurance coefficients and compare them to their empirical counterpart.

## 1.4 Results

As a benchmark we choose the additive separable preference specification, as it enjoys most popularity in the literature and permits an intuitive interpretation of the estimation results. We present results for non-separable preferences in section 1.4.5.

Since the tightness of the borrowing constraints determines the extent to which the assumption of interior allocations is violated we consider both the case of a very tight borrowing constraint, ruling out borrowing altogether (ZBC) and a non-binding natural borrowing constraint (NBBC). All economies are calibrated separately and all feature an operative extensive margin (EM) of female labor supply.

The optimal household policy functions are solved numerically using a policy function

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<sup>14</sup>BPS also assume that transitory wage shocks have no wealth effect on consumption or labor, and assumption that is not exactly correct in a life cycle model, and potentially severely not true near retirement where permanent and transitory shocks become indistinguishable. Finally, of course a first order approximation is just that, and might not capture fully the nonlinear relationship between the wage shocks and endogenous consumption and labor supply decisions.



iteration algorithm with the endogenous grid method proposed by Carroll (2006). Details about the numerical method are provided in Appendix 1.E. The policy functions are then used to simulate a panel of 50,000 households from age 21 to age 80. The results we report below are based on these simulated data.

### 1.4.1 Life Cycles in the Benchmark Economies

Figure 1.1 presents mean life cycle profiles of consumption, assets, labor supply and labor income in the benchmark economy, for both specifications of the borrowing constraint. In the ZBC economy, mean consumption grows before retirement due to the precautionary saving motive, and continues growing after it because the product of gross interest rate and discount factor,  $\frac{1+r}{1+\delta}$ , is greater than one. The life cycle of mean asset position is single-peaked at retirement, which is common in life-cycle models. Declining mean labor supply and rising mean labor income over the life cycle for both the male and female reflect the wealth effect of the rising trend wages. In the NBBC economy, significant differences from the ZBC economy only exist in the first decade of the life cycles. In particular, young households in the NBBC economy on average attain smoother consumption profile, hold negative asset positions, work less and earn less labor income than those in the ZBC economy. The intuition is straightforward. Because the wage trends are increasing with respect to age, households would want to smooth consumption by working less and borrowing in their early life against the higher future income. However, their ability to do so depends on the tightness of borrowing constraints. In the NBBC economy, households can borrow freely with the risk free bond, so they do not need to work a lot in their early life when wages are low. However in the ZBC economy, the only way for young households to increase their consumption is to work more. But because working generates negative utility, the optimal consumption in the ZBC economy is lower when households are young, and the consumption profile is less smoothed than that of the NBBC economy. After the early

periods of life cycle, most households have accumulated enough assets such that borrowing constraints are no longer binding. Hence the rest parts of the life cycles in the ZBC and NBBC economies look very similar to each other.

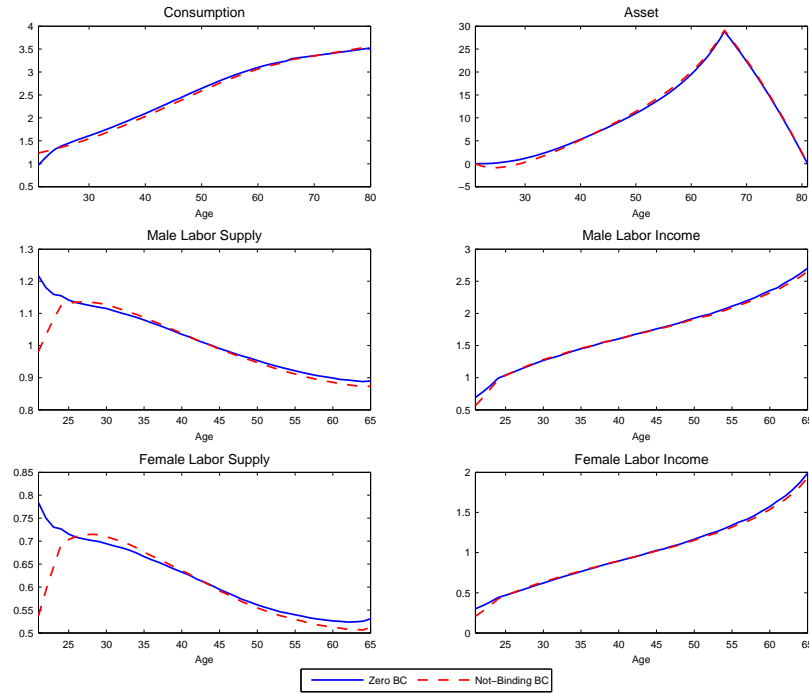


Figure 1.1: Life Cycles of Cross-sectional Means

Figure 1.2 shows the life cycles of income and consumption inequality in these two economies, measured by the cross-sectional variances of log household income and log household consumption. As expected, both income and consumption inequality rise with respect to age due to the accumulation of the permanent wage shocks through the working periods. The rise of consumption inequality slows down for working households near the retirement age, while the rise of household income inequality accelerates during the same period. The consumption inequality is constant after retirement because there is no risk after that time in the model (hence omitted from the graph), and the consumption levels are simply determined by the total discounted wealth of households at retirement. Consistent with the real world, the consumption inequality in the model is smaller than the income

inequality, precisely due to the consumption insurance behaviors. The income inequality in the NBBC economy is notably higher than that of the ZBC economy in the early years of the life cycle. This is due to the fact that in the ZBC economy, young households who suffer from transitory negative wage shocks have to work more to smooth consumption, while the same households in the NBBC economy can simply borrow and avoid increasing labor supply, which results lower income than their counterparts in the ZBC economy. This difference vanishes quickly as households accumulate enough assets through saving, and hence the income inequality patterns are almost identical for older households.

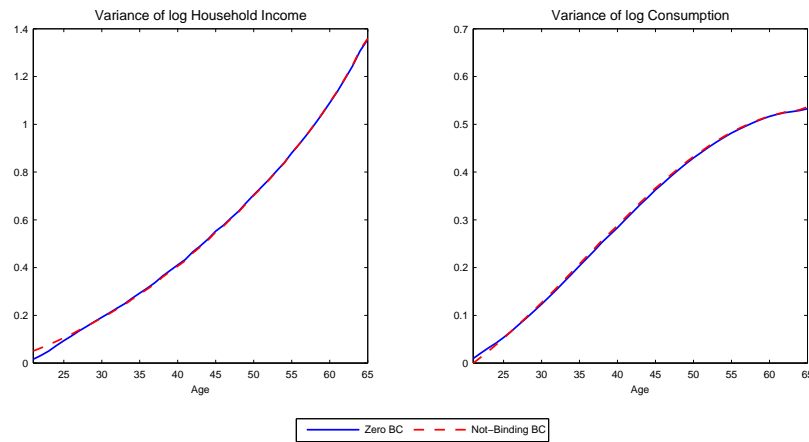


Figure 1.2: Life Cycles of Income and Consumption Inequality

## 1.4.2 Transmission Coefficients in the Model and the Data

The main interest of this paper resides with how household consumption and labor income respond to unexpected permanent and transitory wage shocks, which is captured by the transmission coefficients,  $\kappa$ 's, in Equation (1.3.1). In the model, because the data are simulated, realizations of all wage shocks are known in addition to household consumption and labor income. Thus the transmission coefficients can be obtained directly from running OLS regression with Equation (1.3.1). However, when dealing with the real data, the two types of wage shocks are not observed separately. Consequently the BPS method

is required to obtain estimates of these transmission coefficients from the data. From now on, the results from the simulated data are labeled as “Model True”,<sup>15</sup> while the results estimated from the real data by BPS are labeled as “Data BPS”. We want to compare the “Model True” and “Data BPS” transmission coefficients as a test of how close the model is to the true data generating process (DGP) in the data. Under the assumption that the BPS method works well in estimating the true transmission coefficients in the data, which we verify later in Section 1.5, if the “Model True” transmission coefficients are far away from the “Data BPS” results, it implies that our DGP, i.e., our model, is very different from the true DGP and probably is missing some important components or mechanisms; if the opposite is true, it provides evidence that our model is a good summary of the true DGP, at least for those aspects related to the consumption and labor income dynamics.<sup>16</sup>

Table 1.1 reports the “Model True” transmission coefficients in the two benchmark economies together with the “Data BPS” results.<sup>17</sup> Because BPS only use data of households aged between 30 and 57, the “Model True” transmission coefficients are also acquired for the same group of households in the simulated data.

Comparing the transmission coefficients to consumption in the ZBC economy with the BPS estimates, the consumption insurance in the model seems reasonably close to that in the data. About 1.6% of male and 1.1% of female transitory wage shocks pass through to household consumption, which indicates almost perfect consumption insurance

<sup>15</sup>Due to the large sample size of simulated data, statistic errors are essentially zero. Hence the results from simulated data can be seen as the true values implied by the model.

<sup>16</sup>It may be helpful to point out why it is possible for our model to generated different transmission coefficients from the BPS estimates given the way our model is calibrated. For example, from equation (1.3.2), the BPS estimate of  $\kappa_{c,v_1}$  depends on Frisch elasticities,  $\pi_t$  and  $s_{j,t}$ . Because we calibrate the model to match the BPS Frisch elasticities, no additional differences can be generated from this source other than the biases of the BPS method. However, the joint distribution of  $\pi_t$  and  $s_{j,t}$  are determined by the saving and labor supply behaviors of households, and BPS take them directly from the data, i.e., the true DGP. Hence if the behaviors of households in our model are unrealistic, the model would not get  $\pi_t$  and  $s_{j,t}$  close to the data, hence could have very different  $\kappa_{c,v_1}$ .

<sup>17</sup>We take the BPS results without additive separability assumption and ignoring taxes as our comparison targets.

Table 1.1: Transmission Coefficients in the Data and the Model

	Data BPS	Model True	
		ZBC	NBBC
$\hat{\kappa}_{c,u_1}$	-0.14(0.07)	0.016	0.015
$\hat{\kappa}_{c,u_2}$	-0.06(0.08)	0.011	0.010
$\hat{\kappa}_{c,v_1}$	0.37(0.04)	0.409	0.410
$\hat{\kappa}_{c,v_2}$	0.21(0.03)	0.282	0.281
$\hat{\kappa}_{y_1,u_1}$	1.53(0.12)	1.477	1.478
$\hat{\kappa}_{y_1,u_2}$	0.10(0.05)	-0.009	-0.008
$\hat{\kappa}_{y_1,v_1}$	0.94(0.09)	1.163	1.163
$\hat{\kappa}_{y_1,v_2}$	-0.21(0.03)	-0.226	-0.225
$\hat{\kappa}_{y_2,u_1}$	0.20(0.11)	-0.017	-0.016
$\hat{\kappa}_{y_2,u_2}$	1.90(0.22)	1.829	1.833
$\hat{\kappa}_{y_2,v_1}$	-0.77(0.13)	-0.473	-0.474
$\hat{\kappa}_{y_2,v_2}$	1.39(0.09)	1.353	1.353

Note: The numbers inside parentheses are standard errors taken from BPS.

against transitory wage shocks. This result is common in life-cycle models with saving and additive separable preference. With regarding to the permanent ones, about 40.9% of male and 28.2% of female permanent wage shocks pass through to household consumption. The amount of consumption insurance against permanent wage shocks implied by these numbers is quite high in the consumption insurance literature. On the other hand, the BPS estimates of the pass-through rates of transitory wage shocks are negative, -14% for male and -6% for female, but not significant different from the close-to-zero “Model True” results of  $\hat{\kappa}_{c,u_j}$  given the BPS standard errors. The BPS estimates of the pass-through rates of permanent wage shocks are 37% for male and 21% for female, which are slightly lower than what the model implies. This reveals a slightly higher amount of consumption insurance against permanent wage shocks in the data. Given the simplicity of the model, it is a prominent result that the model can already account for about 94% and 91% of consumption insurance against male and female permanent wage shocks observed in the data.

Not only does the model fit well the consumption insurance pattern in the data, it also

generates similar labor income responses to wage shocks as the data suggest. In the ZBC economy, the transmission coefficients to male and female labor income from their own transitory wage shocks,  $\kappa_{y_j, u_j}$ , are larger than one which means that labor supply increases when wages are temporarily high.<sup>18</sup> This is because the transitory shocks have small wealth effect, and the substitution effect dominates. The transmission coefficients to labor income from the other earner's transitory wage shocks,  $\kappa_{y_j, u_{-j}}$ , are slightly negative, which are merely consequences of the insignificant wealth effects of transitory shocks. The transmission coefficients to male and female labor income from their own permanent wage shocks,  $\kappa_{y_j, v_j}$ , are closer to one than those from their transitory shocks, which means labor supply on average responds less to its own permanent wage shocks. On the contrary, both male and female labor supply respond more actively to the other earner's permanent wage shocks. The values of  $\kappa_{y_j, v_{-j}}$  represent a 0.226% fall of male labor supply with respect to a 1% increase of female permanent wage, and a 0.473% fall of female labor supply with respect to the same 1% shock to male permanent wage. These results suggest that a main labor supply adjustment of households when facing a permanent wage shock to one earner is to change the other earner's labor supply and reduce the shock's impact to household income. Female labor supply appears to be more sensitive to all kinds of wage shocks with larger absolute percentage changes than male labor supply. Comparing the transmission coefficients to labor income in the ZBC economy with the "Data BPS" counterparts, it seems that the model is able to reproduce the major patterns in the data, and the differences are mostly not statistically significant.

It does not seem to matter for the transmission coefficients whether borrowing constraints are calibrated as tight or relaxed because they are almost identical in the ZBC and NBBC benchmark economies. However, this result does not indicate that the tightness of

<sup>18</sup>The labor supply responses to wage shocks can be deduced by subtracting the percentage changes of wages from the corresponding transmission coefficients to labor income.

borrowing constraints is irrelevant to consumption insurance. There are mainly two causes of the identical transmission coefficients here. First, since only households aged from 30 to 57 are used in the calculation of the transmission coefficients, the young households who are mostly affected by the tightness of borrowing constraints are excluded. Second, the two benchmark economies are calibrated separately to fit the same empirical targets. If all the parameters of the ZBC economy are kept the same but only the borrowing constraints are relaxed, the amount of consumption insurance would change. But then some features of the economy would become counterfactual such as the aggregate asset-income ratio. Once recalibrated, the original amount of consumption insurance is restored.

### 1.4.3 Age Profiles of Transmission Coefficients

The transmission coefficients in Table 1.1 are sample averages for all the households aged from 30 to 57. It is natural that households at different stages of life cycle could respond differently to the same wage shocks. To understand this, it is useful to look at the age profiles of these transmission coefficients.

Figure 1.3 presents all the age profiles of transmission coefficients in the two benchmark economies. The age profiles are again almost identical between the ZBC and NBBC economies. The transmission coefficients to consumption from transitory shocks,  $\kappa_{c,u_j}$ , are small, positive, and relatively stable over the life cycle. This is simply reflecting the small wealth effect of transitory shocks. With regard to permanent wage shocks, the transmission coefficients to consumption,  $\kappa_{c,v_j}$ , decreases with age, i.e., the amount of consumption insurance against permanent shocks rises over the life cycle. This is the result of the increasing asset holding and decreasing human wealth with age, and hence wage shocks become less important to old households. BPS also find an increasing age profile of total consumption insurance against permanent male wage shocks in the data, which is another evidence suggesting a good fit between the model and the data.

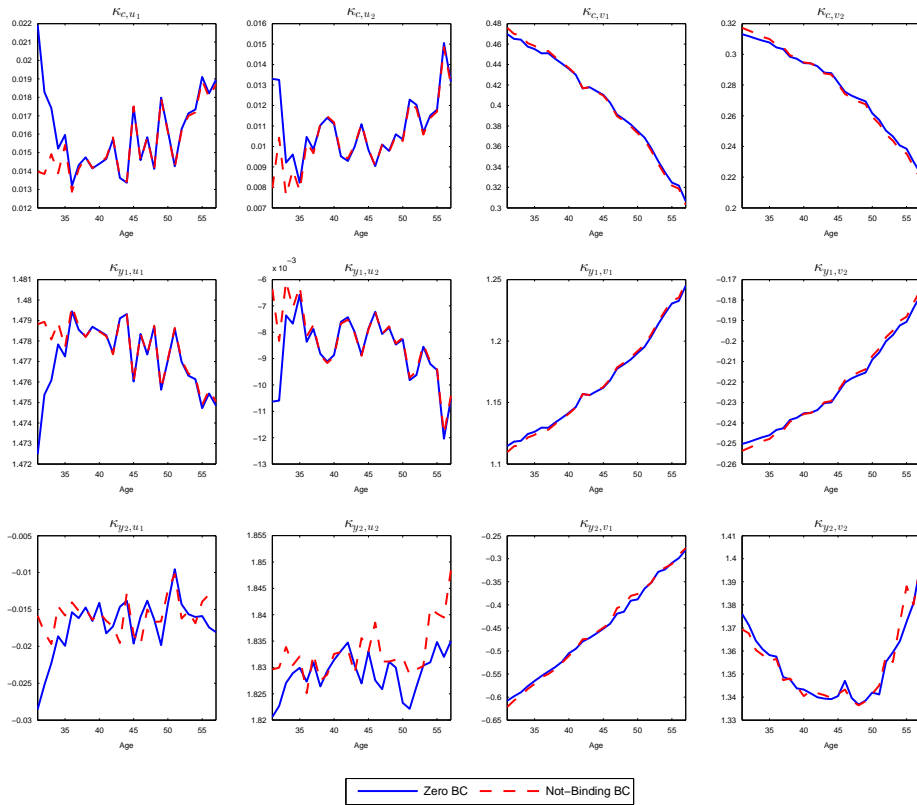


Figure 1.3: Age Profiles of Transmission Coefficients



The age profiles of the transmission coefficients to labor income from transitory wage shocks,  $\kappa_{y_j, u_j}$  and  $\kappa_{y_j, u_{-j}}$ , can also be explained by the small wealth effect of transitory shocks. The transmission coefficients to male labor income from his own permanent wage shocks,  $\kappa_{y_1, v_1}$ , is larger for old households because the wealth effect of a permanent wage shock is smaller for them, and thus the substitution effect dominates. This means young households increase their male labor supply less than the old households in response to a positive male permanent wage shocks. On the other hand for the female,  $\kappa_{y_2, v_2}$  is quite stable over the life cycle. The transmission coefficients to one earner's labor income from the other earner's permanent wage shocks,  $\kappa_{y_j, v_{-j}}$ , declines in absolute value with age because older households can smooth consumption with their savings and rely less on adjustments of labor supply.

#### 1.4.4 Counterfactuals and Decomposition of Consumption Insurance

There are mainly three consumption insurance mechanisms in the model: (1) self-insurance through trading the risk free bond; (2) within-household risk sharing due to the non-zero imperfectly correlated income of the two earners; and (3) the endogenous labor supply.<sup>19</sup> Trading the risk free bond smooths consumption by transferring household income across time. Within-household risk sharing reduces the impact of a wage shock to one earner because the other earner's income dilutes the effect of the wage shock to household income. Finally, the endogenous labor supply allows earners to change their labor supply in response to a wage shock, which results a different percentage change of household income. Consumption insurance in the model can also be decomposed by its sources: it can be provided by (1) trading the risk free bond, (2) the male earner and (3) the female earner.

<sup>19</sup>The endogenous extensive margin of female labor supply serves as another insurance mechanism. However, experiments show that the impact of it to consumption insurance in the model is very small because it is only at work when the female labor supply changes between zero and positive. It also causes the problem of undefined log income change. Hence it is not discussed here, and the results in this subsection are from models without the extensive margin of female labor supply.

One source of consumption insurance can work with more than one insurance mechanisms. For example, the female earner provides insurance against male wage shocks through both the within-household risk sharing and the endogenous labor supply mechanisms.

Equipped with the model in hand, many questions of immediate interest can be addressed. For example, how much consumption insurance remains if some insurance mechanisms are not available? What are the contributions of different sources to the total consumption insurance? How do the contributions of different sources evolve as more insurance mechanisms become available and over the life cycle? To answer these questions, we compute the transmission coefficients in a bunch of counterfactual economies with different combinations of the consumption insurance mechanisms based on the parametrization of the benchmark economies. We focus on the consumption insurance against the male wage shocks because they are the most important shocks to household income, and also the patterns against the female wage shocks should be qualitatively similar.

Table 1.2 shows how a 1% male wage shock, either permanent or transitory, transmits into household consumption in a sequence of economies with ZBC. The results with NBBC would be very similar. The economies from (i) to (iv) can be seen as generated by adding one additional insurance mechanism at a time. It also shows a decomposition of the amount of insurance provided by the three different sources.

Table 1.2: Transmission of a 1% Male Wage Shock

Economy by Household (HH) Structure	% Change of				Insurance Provided by			
	Male Income	Female Income	Household Income	Household Consumption	Male Earner	Female Earner	Risk Free Bond	Total Insurance
<i>A. Permanent Shock</i>								
(i) 1-earner HH with exogenous labor	1	–	1	0.711	0	–	28.9%	28.9%
(ii) 1-earner HH with endogenous labor	0.992	–	0.992	0.623	0.8%	–	36.9%	37.7%
(iii) 2-earner HH with endogenous male and exogenous female labor	1.103	0	0.743	0.484	–10.3%	36.0%	25.9%	51.6%
(iv) 2-earner HH with endogenous labor of both earners	1.161	–0.569	0.517	0.411	–16.1%	64.4%	10.6%	58.9%
<i>B. Transitory Shock</i>								
(i) 1-earner HH with exogenous labor	1	–	1	0.029	0	–	97.1%	97.1%
(ii) 1-earner HH with endogenous labor	1.469	–	1.469	0.026	–46.9%	–	144.3%	97.4%
(iii) 2-earner HH with endogenous male and exogenous female labor	1.475	0	0.994	0.019	–47.5%	48.1%	97.5%	98.1%
(iv) 2-earner HH with endogenous labor of both earners	1.477	–0.022	0.928	0.016	–47.7%	54.9%	91.2%	98.4%

Note: The results are based on sample averages of transmission coefficients from different economies with the same parameters as the ZBC benchmark economy but different household structures.

With regard to permanent shocks, the main takeaway from these results is that while

households can attain some amount of consumption insurance against male permanent wage shocks without the female earner, the female earner, if present, increases the total insurance significantly and is the dominating source of such insurance. Economy (i) corresponds to the standard incomplete-markets model with exogenous income shocks investigated by Kaplan and Violante (2010), and all the insurance is provided by trading the risk free bond.<sup>20</sup> In Economy (ii), the labor supply of the single earner is now endogenous, and it provides only 0.8% reduction of permanent shocks by transforming a 1% wage shock into a 0.992% income shock. The rest of insurance is provided by trading the risk free bond, and the total insurance only rises from 28.9% to 37.7%. This suggests that the endogenous labor of a single male earner is not a very effective insurance mechanism against his own permanent wage shocks. When the female earner with exogenous labor supply is added in Economy (iii), the total amount of insurance increases more by an additional 13.9%. And it is evident as shown in the decomposition that households are shifting the task of consumption insurance to the female earner (or the within-household risk sharing mechanism), and rely less on the other two sources. Actually, the male earner now provides negative insurance by increasing his labor supply in response to a positive male permanent wage shock. Finally, in Economy (iv), when the labor supply of the female earner is also endogenous, about  $\frac{64.4\%}{58.9\%} \approx 109\%$  of the total consumption insurance is provided by the female earner. The insurance provided by the male earner becomes more negative and the role of trading risk free bond further diminishes. Note that the significant increase of insurance provided by the female earner from Economy (iii) to (iv) reveals that the behavior response of female labor supply is an important contributing factor which makes this source so preferred by households. With regard to transitory shocks, the total consumption insurance is almost

<sup>20</sup>Note that the amount of consumption insurance against permanent shocks here is a little higher than that in Kaplan and Violante (2010) (29% v.s. 23%). This is because the model here in this case is counterfactual and generates too much saving. Also we only use age 30 to 57 households here while Kaplan and Violante (2010) results include the very young households at the beginning of their life cycles.

perfect in all the four economies. Trading the risk free bond appears to always offer most of the insurance, and the endogenous male labor supply amplifies the shocks rather than reducing them due to the substitution effect.

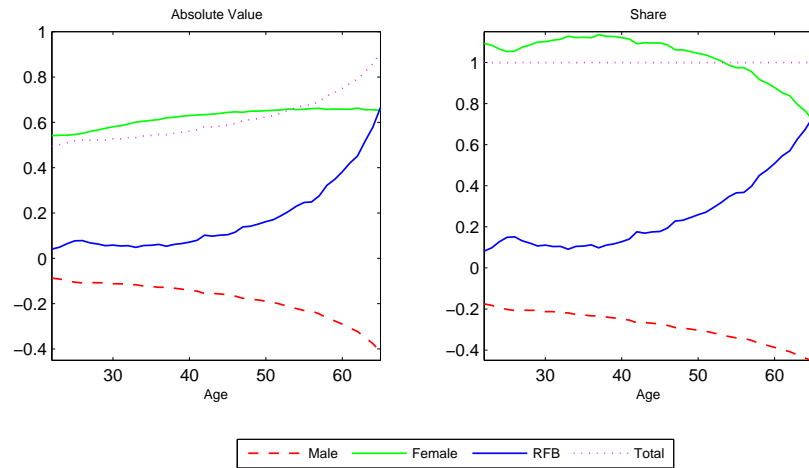


Figure 1.4: Age Profiles of Consumption Insurance Decomposition

To investigate the age profiles of consumption insurance from these three sources against permanent male wage shocks, the same decomposition is conducted for each cohort in Economy (iv), and the results are plotted in Figure 1.4 in terms of both values of insurance and shares of total insurance. The contribution of the female earner does not vary a lot over the life cycle, and is the largest among the three components on average. The contribution of the male earner is negative and decreases with age, which means the substitution effect dominates the wealth effect over the life cycle. The insurance provided by trading the risk free bond keeps rising over the life cycle, which suggests that old households rely more on the self-insurance through saving. This is also the main reason for the increasing age profile of total consumption insurance.

### 1.4.5 Non-separable Preference

The BPS estimates of Frisch elasticities without the separability assumption imply Frisch complementarity between consumption and leisure ( $\eta_{c,w_j} < 0$ ,  $\eta_{h_j,p} > 0$ ), and also between leisures of the spouses ( $\eta_{h_j,w_{-j}} > 0$ ). However, none of these Frisch cross-elasticities are significant at 1% level. So we think the additive separable preference is still a reasonable modeling choice for preference, and as Table 1.1 shows, it is capable of generating the consumption and labor income dynamics very close to what the data imply. On the other hand, using non-separable preference could, in principle, allow the model to fit better more aspects of the data, for example, the negative response of consumption with respect to positive transitory wage shocks ( $\kappa_{c,u_j} < 0$ ) in the BPS results. The main difficulty of using a non-separable preference is how to find, if it exists at all, a functional form that can match the Frisch elasticity targets while keeping the model computationally tractable. While we do not have a perfect solution to this difficulty, we take a first step by proposing a non-separable preference as in Equation (1.2.1) which is tractable and can capture the Frisch complementarities implied by the BPS results. We want to learn how such non-separability affects the amount of consumption insurance and other transmission coefficients in the model, and to what extent it can help our model to match the data better.

To save space, we only discuss here the sample averages of the transmission coefficients with the non-separable preference in two economies augmented with ZBC and NBBC. Table 1.3 has the relevant results. All the other previous results for the additive separable preference have their counterparts for the non-separable preference, and they are available in Appendix 1.A.

In terms of consumption insurance in the ZBC economy, the main difference with the non-separable preference is that consumption and leisure are now Frisch complements which means higher labor supply reduces the marginal utility of consumption, and higher consumption increases the marginal disutility of labor supply. As a result, the consump-

Table 1.3: Transmission Coefficients in the Data and the Model  
(Non-separable Preference)

	Data BPS	Model True	
		ZBC	NBBC
$k_{c,u_1}$	-0.14(0.07)	-0.129	-0.136
$\kappa_{c,u_2}$	-0.06(0.08)	-0.076	-0.081
$\kappa_{c,v_1}$	0.37(0.04)	0.275	0.274
$\kappa_{c,v_2}$	0.21(0.03)	0.180	0.179
$\kappa_{y_1,u_1}$	1.53(0.12)	1.948	1.965
$\kappa_{y_1,u_2}$	0.10(0.05)	0.086	0.093
$\kappa_{y_1,v_1}$	0.94(0.09)	1.069	1.074
$\kappa_{y_1,v_2}$	-0.21(0.03)	-0.470	-0.471
$\kappa_{y_2,u_1}$	0.20(0.11)	0.117	0.133
$\kappa_{y_2,u_2}$	1.90(0.22)	1.907	1.928
$\kappa_{y_2,v_1}$	-0.77(0.13)	-0.615	-0.624
$\kappa_{y_2,v_2}$	1.39(0.09)	1.202	1.211

Note: The numbers inside parentheses are standard errors taken from BPS.

tion responses to transitory wage shocks are now negative in the model because transitory shocks have little wealth effect and labor supply moves in the same direction as the transitory shocks. For the permanent wage shocks, the same logic applies, and hence the consumption response to permanent wage shocks are lowered by this complementarity. However, due to the wealth effect of permanent shocks, the overall consumption responses to permanent wage shocks remain positive. Compare with the “Data BPS” results, the model now matches well the more than 100% consumption insurance against transitory shocks, and actually generates higher amount of consumption insurance against permanent wage shocks than the BPS estimates. The pass-through rates of permanent wage shocks to consumption in the model are 27.5% and 18.0% for male and female permanent wage shocks, respectively. The model with non-separable preference matches better the consumption insurance against female permanent wage shocks than the model with additive separable preference but matches worse that against male permanent shocks.

Because of the Frisch complementarity between leisures of the spouses, the labor in-

come responses to the other earner's transitory wage shocks are positive, which is different from the additive separable preference case. Overall, the transmission coefficients to labor income from wage shocks in the model replicate well what happens in the data. Most of the differences are not statistically significant. The only two exceptions are  $\kappa_{y_1, u_1}$  and  $\kappa_{y_1, v_2}$ , which are further away from their counterparts in the data measured in units of their BPS standard errors. This is mainly due to the counterfactually high Frisch elasticity of male labor supply with respect to his own wage,  $\eta_{h_1, w_1}$ , in the model which is a result of the calibration difficulty with non-separable preference explained in Section 1.2.3. The transmission coefficients in the NBBC economy are again very close to those of the ZBC economy, the reasons of which have already been discussed in the additive separable preference case.

## 1.5 Performance of the BPS Method

Section 2.4 shows that the Bewley type model with two-earner households and endogenous labor supply is a good model for household behaviors when facing unexpected wage shocks because the responses of household consumption and labor income in the model match the empirical results from BPS well. This conclusion would not be valid if the BPS estimates were significantly biased. To address such concern, we apply the BPS estimation method to the simulated data from the model, and label those results as "Model BPS". Note that because the sample size of simulated data is very large, the standard errors of the "Model BPS" results are practically zero, and hence a comparison between the "Model True" and "Model BPS" results reveals only the biases of the BPS method and can tell us how well the BPS method identify these parameters. Following the BPS baseline results, we do not estimate the outside insurance  $\beta$  and set it at the true value zero unless specified

otherwise.<sup>21</sup> Details of how we implement the BPS method are available in Appendix 1.D.

We find that, for both the additive separable and non-separable preferences, the BPS method works well in the estimation of the Frisch elasticities and the transmission coefficients if implemented properly, and the biases are in general reasonably small. Hence the previous conclusions drawn from the comparison between the model-based and the BPS empirical results on consumption insurance and transmission coefficients are still valid. The estimation of the consumption Frisch elasticity  $\eta_{c,p}$  is sensitive to the moment conditions used, but can be improved significantly by using multiple moment conditions and iterated GMM method with updated weighting matrices. The estimation of the outside insurance  $\beta$ , if allowed, is unstable and significant biased, and hence should be avoided. The extensive margin of female labor supply induces slightly larger biases on female related parameters, while the tightness of borrowing constraints has almost no impacts on the performance of the BPS method due to the sample selection criterion on household age.

### 1.5.1 With Additive Separable Preference

We first report the results based on the two benchmark economies with the additive separable preference. In this case, the separability assumption can be imposed and all the cross Frisch elasticities are assumed to be zero. As an intermediate step to the transmission coefficients, other Frisch elasticities still need to be estimated first. Table 1.4 has the relevant results. The estimation of the labor Frisch elasticities,  $\eta_{h_j,w_j}$ , is very accurate, with slightly larger bias on  $\eta_{h_2,w_2}$  due to the extensive margin of female labor supply. The biases on the estimation of the consumption Frisch elasticity,  $\eta_{c,p}$ , are also small but are sensitive to the moment conditions used which we discuss later in more details. The tightness

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<sup>21</sup>Although the model is featured with a social insurance system in the form of the retirement benefit  $b$ , the effect of this is accounted for explicitly in the modified BPS formulas of this paper by  $q_t$ . Hence the true value of the external insurance coefficient  $\beta$  in the model should be zero. Relevant details are in Appendix 1.D.



of borrowing constraints does not seem to affect the biases a lot because the sample only includes simulated households aged from 30 to 57, and young households who are mostly affected by the tight borrowing constraints are not included in the estimation.

Table 1.4: Estimation of Frisch Elasticities and External Insurance Coefficient

	Model True	Model BPS	
		ZBC	NBBC
$\eta_{c,p}$	0.613	0.610	0.608
$\eta_{h_1,w_1}$	0.490	0.490	0.490
$\eta_{h_2,w_2}$	0.849	0.881	0.882

Table 1.5 reports the “Model True” and “Model BPS” results of transmission coefficients. In terms of consumption insurance, the BPS method overestimates the consumption insurance against transitory wage shocks and male permanent wage shocks, but underestimates that against female permanent shocks. However, the magnitudes of the biases are reasonably small. The BPS estimates of  $\kappa_{c,u_j}$  must be zero with the separability assumption and the assumption that transitory shocks do not have wealth effect. The second assumption is likely to be the cause of the downward biases here because for finitely lived households, even transitory shocks have small wealth effect especially for old households. For  $\kappa_{c,v_j}$ , the exact causes of the biases are harder to identify.<sup>22</sup> The age profiles of “Model True” and “Model BPS”  $\kappa_{c,v_j}$  are plotted in Figure 1.5. The biases on  $\kappa_{c,v_1}$  is decreasing with age, while the biases on  $\kappa_{c,v_2}$  is increasing. For the transmission coefficients to labor income, the biases of the BPS method are generally small except for the transmission coefficients to female labor income from permanent wage shocks, i.e.,  $\kappa_{y_2,v_j}$ . The BPS method overestimates a significant amount of female labor responses to the permanent shocks. The reason for such larger biases is likely to be the selection problem caused by the extensive margin of female labor supply. Again, the impacts of the tightness of borrowing constraints are negligible.

<sup>22</sup>It is interesting that the biases on  $\kappa_{c,v_1}$  and  $\kappa_{c,v_2}$  have different signs. Our conjecture is that they are related to the fact that female income is more volatile than male, and the way BPS method approximates  $s_{j,t}$ .

Table 1.5: Estimation of Transmission Coefficients

	ZBC		NBBC	
	Model True	Model BPS	Model True	Model BPS
$k_{c,u_1}$	0.016	0	0.015	0
$\kappa_{c,u_2}$	0.011	0	0.010	0
$\kappa_{c,v_1}$	0.409	0.386	0.410	0.386
$\kappa_{c,v_2}$	0.282	0.310	0.281	0.308
$\kappa_{y_1,u_1}$	1.477	1.490	1.478	1.490
$\kappa_{y_1,u_2}$	-0.009	0	-0.008	0
$\kappa_{y_1,v_1}$	1.163	1.180	1.163	1.179
$\kappa_{y_1,v_2}$	-0.226	-0.249	-0.225	-0.249
$\kappa_{y_2,u_1}$	-0.017	0	-0.016	0
$\kappa_{y_2,u_2}$	1.829	1.881	1.833	1.882
$\kappa_{y_2,v_1}$	-0.473	-0.557	-0.474	-0.560
$\kappa_{y_2,v_2}$	1.353	1.433	1.353	1.435

When estimating the consumption Frisch elasticity  $\eta_{c,p}$ , we find that the result is sensitive to the moment conditions and the weighting matrix used in the GMM system. We also find that the outside insurance  $\beta$  is not well identified by these moment conditions if estimated jointly with  $\eta_{c,p}$ .<sup>23</sup> Table 1.6 reports the results of a sensitivity analysis which shows how different moment conditions and weighting matrices affect the estimation of  $\eta_{c,p}$ ,  $\beta$ , and the transmission coefficients to consumption from permanent wage shocks  $\kappa_{c,v_j}$ . The moment conditions related to consumption growth  $\Delta c_t$  tend to overestimate  $\eta_{c,p}$ , while those related to male labor income growth  $\Delta y_{1,t}$  tend to underestimate  $\eta_{c,p}$ .<sup>24</sup> The higher the estimated  $\eta_{c,p}$ , the lower the estimated consumption insurance against permanent shocks. We do not use the moment conditions related to female labor income here to avoid the selection problem of female extensive margin. The one-step GMM uses the identity weighting matrix and combine all the five moment conditions, while the iterated

<sup>23</sup>We first estimate all the other Frisch elasticities and wage variances using a group of just identified moment conditions which do not involve  $\eta_{c,p}$  and  $\beta$ . The estimation in this step is very accurate. Then conditional on these estimates, we estimate  $\eta_{c,p}$  (and  $\beta$ ) using the second order moment conditions of consumption growth, labor income growth and wage growth. More details are in Appendix 1.D

<sup>24</sup>This shows clearly that the BPS formulas are biased. Because if not, different moment conditions should all identify the same true value of  $\eta_{c,p}$ .

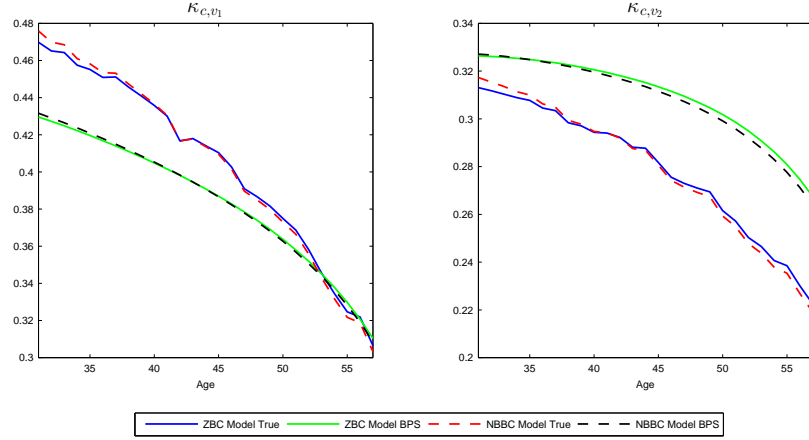


Figure 1.5: Age Profiles of  $\kappa_{c,v_j}$ : Model True vs. Model BPS

GMM uses the same moment conditions but updates the weighting matrix optimally based on the estimation results from last step until convergence. For the additive separable preference, these two methods deliver very close estimate of  $\eta_{c,p}$ , but it is no longer the case for the non-separable preference which we report later in Table 1.9. The original BPS paper uses the one-step GMM method with the identity weighting matrix, but we find the iterated GMM method is more stable and accurate, so the estimation results in this paper are based on this method. If we also want to estimate the outside insurance  $\beta$ , the estimation of  $\eta_{c,p}$  does not change a lot. However, the estimation of  $\beta$  is not only severely biased but also sensitive to the weighting matrix used. This suggests that the identification of  $\beta$  is very weak from these moment conditions.

### 1.5.2 With Non-separable Preference

When the non-separable preference is the case, the separability assumption is no longer appropriate. Therefore all the Frisch elasticities need to be estimated. The estimation results are now based on the simulated data from the two economies with the non-separable preference. Table 1.7 presents the “Model True” and “Data BPS” results for the Frisch

Table 1.6: Sensitivity Analysis for the Estimation of  $\eta_{c,p}$  and  $\beta$

Moment/Method	$\eta_{c,p}$	$\beta$	$\kappa_{c,v_1}$	$\kappa_{c,v_2}$
<i>A. Model True Values</i>				
	0.613	0	0.409	0.282
<i>B. Estimate only <math>\eta_{c,p}</math> with single moment condition</i>				
$E(\Delta c_t^2)$	0.770	—	0.426	0.342
$E(\Delta y_{1,t}^2)$	0.496	—	0.350	0.281
$E(\Delta c_t \Delta y_{1,t})$	0.634	—	0.392	0.315
$E(\Delta c_t \Delta w_{1,t})$	0.764	—	0.424	0.341
$E(\Delta y_{1,t} \Delta w_{1,t})$	0.491	—	0.348	0.280
<i>C. Estimate only <math>\eta_{c,p}</math> with all the five moment conditions</i>				
One-step GMM	0.606	—	0.385	0.309
Iterated GMM	0.610	—	0.386	0.310
<i>D. Estimate <math>\eta_{c,p}</math> and <math>\beta</math> jointly with all the five moment conditions</i>				
One-step GMM	0.604	-0.217	0.426	0.342
Iterated GMM	0.611	0.188	0.343	0.275

elasticities.<sup>25</sup> The biases are larger for those Frisch elasticities related to the female due to the extensive margin of female labor supply. Table 1.8 reports the results on transmission coefficients. In general, the biases on the transmission coefficients are small, and have the same signs as those with the additive separable preference. The tightness of borrowing constraints do not matter much to the biases.

Table 1.9 reports the sensitivity results for the estimation of  $\eta_{c,p}$  (and  $\beta$ ) with the non-separable preference. The patterns are similar to those with the additive separable preference, and it is now obvious that the one-step GMM method performs worse than the iterated GMM method.

<sup>25</sup>The “Model True” Frisch elasticities are no longer deep parameters with the non-separable preference, and depend on allocations. The ones reported are sample averages of household level elasticities calculated using formulas in Appendix 1.C.

Table 1.7: Estimation of Frisch Elasticities and External Insurance Coefficient  
(Non-separable Preference)

	ZBC		NBBC	
	Model True	Model BPS	Model True	Model BPS
$\eta_{c,p}$	0.373	0.362	0.369	0.360
$\eta_{c,w_1}$	-0.148	-0.149	-0.150	-0.152
$\eta_{c,w_2}$	-0.086	-0.100	-0.088	-0.104
$\eta_{h_1,p}$	0.223	0.204	0.215	0.199
$\eta_{h_1,w_1}$	0.988	0.991	0.996	1.001
$\eta_{h_1,w_2}$	0.110	0.129	0.110	0.135
$\eta_{h_2,p}$	0.223	0.192	0.215	0.190
$\eta_{h_2,w_1}$	0.188	0.121	0.189	0.130
$\eta_{h_2,w_2}$	0.910	0.983	0.917	0.996

## 1.6 Conclusions

We have shown that a Bewley type model with two-earner households and endogenous labor supply decisions can generate the amount of consumption insurance consistent with what BPS have estimated on the US data. This suggests that life-cycle models augmented with these two features should be used in the studies where the consumption insurance mechanisms are important. Moreover, policy evaluations related to household structure and labor supply such as the labor income tax rates and unemployment benefits, etc., should take into consideration the roles of them in providing consumption insurance. The good fit of labor income responses in the model also suggest that it may be applied to issues related to household labor supply decisions. We have also shown that the BPS method can provide important information about how consumption and labor income respond to wage shocks in the data. Applications of this method to other data sets may provide more robust or cross-country evidences on consumption insurance, and help us understand better the economic stories behind.

Table 1.8: Estimation of Transmission Coefficients  
(Non-separable Preference)

	ZBC		NBBC	
	Model True	Model BPS	Model True	Model BPS
$k_{c,u_1}$	-0.129	-0.149	-0.136	-0.152
$k_{c,u_2}$	-0.076	-0.100	-0.081	-0.104
$k_{c,v_1}$	0.275	0.270	0.274	0.269
$k_{c,v_2}$	0.180	0.181	0.179	0.180
$k_{y_1,u_1}$	1.948	1.991	1.965	2.001
$k_{y_1,u_2}$	0.086	0.129	0.093	0.135
$k_{y_1,v_1}$	1.069	1.085	1.074	1.089
$k_{y_1,v_2}$	-0.470	-0.480	-0.471	-0.480
$k_{y_2,u_1}$	0.117	0.121	0.133	0.130
$k_{y_2,u_2}$	1.907	1.983	1.928	1.996
$k_{y_2,v_1}$	-0.615	-0.766	-0.624	-0.769
$k_{y_2,v_2}$	1.202	1.388	1.211	1.390

Table 1.9: Sensitivity Analysis For the Estimation of  $\eta_{c,p}$  and  $\beta$   
(Non-separable Preference)

Moment/Method	$\eta_{c,p}$	$\beta$	$k_{c,v_1}$	$k_{c,v_2}$
<i>A. Model True Values</i>				
	0.373	0	0.275	0.180
<i>B. Estimate only <math>\eta_{c,p}</math> with single moment condition</i>				
$E(\Delta c_t^2)$	0.425	—	0.295	0.198
$E(\Delta y_{1,t}^2)$	0.173	—	0.179	0.120
$E(\Delta c_t \Delta y_{1,t})$	0.376	—	0.275	0.184
$E(\Delta c_t \Delta w_{1,t})$	0.459	—	0.308	0.206
$E(\Delta y_{1,t} \Delta w_{1,t})$	0.217	—	0.203	0.136
<i>C. Estimate only <math>\eta_{c,p}</math> with all the five moment conditions</i>				
One-step GMM	0.277	—	0.232	0.155
Iterated GMM	0.362	—	0.270	0.181
<i>D. Estimate <math>\eta_{c,p}</math> and <math>\beta</math> jointly with all the five moment conditions</i>				
One-step GMM	0.354	-0.574	0.309	0.207
Iterated GMM	0.359	-0.344	0.297	0.199

# Bibliography

- Aiyagari, S. Rao (1994). “Uninsured Idiosyncratic Risk and Aggregate Saving”. In: *The Quarterly Journal of Economics* 109.3, pp. 659–684.
- Attanasio, Orazio, Hamish Low, and Virginia Sánchez-Marcos (2005). “Female Labor Supply as Insurance Against Idiosyncratic Risk”. English. In: *Journal of the European Economic Association* 3.2/3, pp. 755–764.
- Blundell, Richard, Luigi Pistaferri, and Ian Preston (2008). “Consumption Inequality and Partial Insurance”. In: *American Economic Review* 98.5, pp. 1887–1921.
- Blundell, Richard, Luigi Pistaferri, and Itay Saporta-Eksten (2014). *Consumption Inequality and Family Labor Supply*. Working Paper.
- Carroll, Christopher D. (2006). “The method of endogenous gridpoints for solving dynamic stochastic optimization problems”. In: *Economics Letters* 91.3, pp. 312–320.
- Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante (2010). “The Macroeconomic Implications of Rising Wage Inequality in the United States”. English. In: *Journal of Political Economy* 118.4, pp. 681–722.
- (2014). *Optimal Tax Progressivity: An Analytical Framework*. Working Paper 19899. National Bureau of Economic Research.
- Holter, Hans A., Dirk Krueger, and Serhiy Stepanchuk (2014). *How Does Tax Progressivity and Household Heterogeneity Affect Laffer Curves?* Working Paper 20688. National Bureau of Economic Research.

- Huggett, Mark (1993). “The risk-free rate in heterogeneous-agent incomplete-insurance economies”. In: *Journal of Economic Dynamics and Control* 17.5-6, pp. 953 –969.
- Hurst, Erik (2008). *The Retirement of a Consumption Puzzle*. Working Paper 13789. National Bureau of Economic Research.
- Kaplan, Greg and Giovanni L. Violante (2010). “How Much Consumption Insurance beyond Self-Insurance?” In: *American Economic Journal: Macroeconomics* 2.4, pp. 53–87.
- Ortigueira, Salvador and Nawid Siassi (2013). “How important is intra-household risk sharing for savings and labor supply?” In: *Journal of Monetary Economics* 60.6, pp. 650 –666.
- Rupert, Peter and Giulio Zanella (2012). *Revisiting wage, earnings, and hours profiles*. University of California at Santa Barbara, Economics Working Paper Series qt61f2f1hv. Department of Economics, UC Santa Barbara.
- Santaeulàlia-Llopis, Raül and Yu Zheng (2014). *Partial Insurance and Aggregate Welfare in China, 1989-2009*. Working Paper.
- Tauchen, George (1986). “Finite state markov-chain approximations to univariate and vector autoregressions”. In: *Economics Letters* 20.2, pp. 177 –181.

## **1.A Supplementary Results For the Non-separable Preference**

This section presents all the figures and tables for the ZBC and NBBC economies with the non-separable preference which are not included in the main text. Each one of them corresponds exactly to one figure or table in the main text for the additive separable preference.



## 1.A.1 Life Cycles

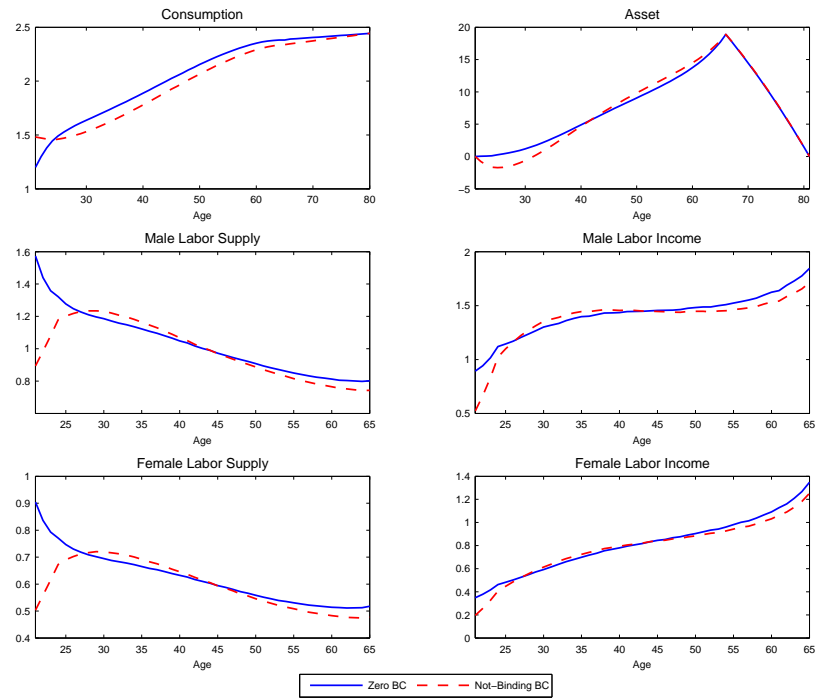


Figure 1.6: Life Cycles of Cross-sectional Means  
(Non-separable Preference)

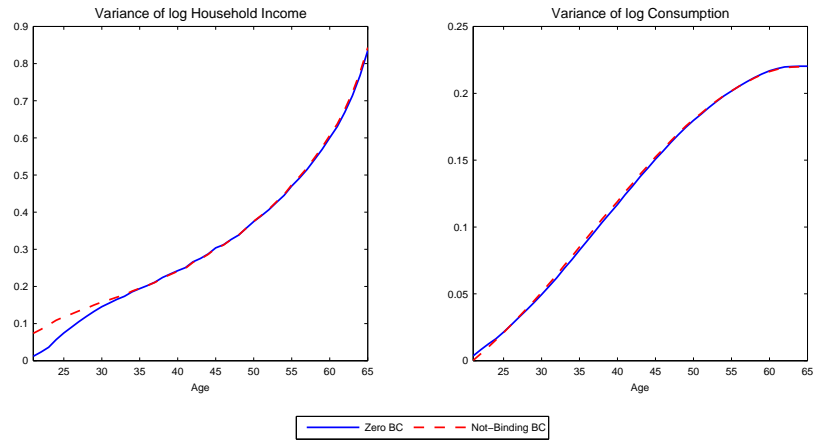


Figure 1.7: Life Cycles of Income and Consumption Inequality (Non-separable Preference)

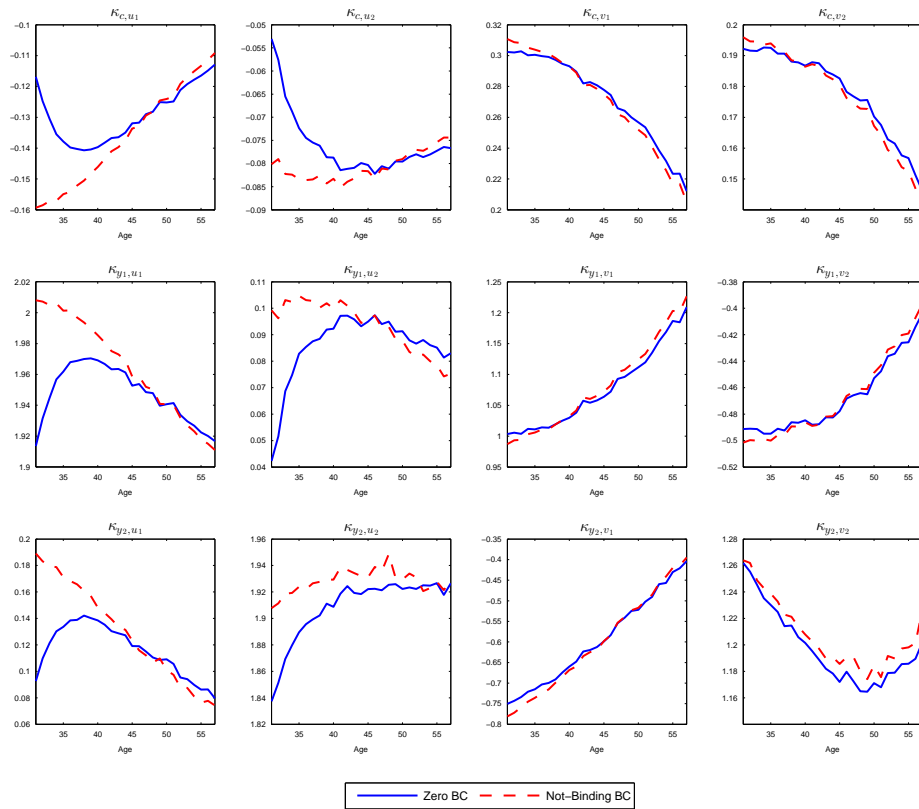


Figure 1.8: Age Profiles of Transmission Coefficients (Non-separable Preference)

## 1.A.2 Counterfactuals and Consumption Insurance Decomposition

Table 1.10: Transmission of a 1% Male Wage Shock  
(Non-separable Preference)

Economy by Household (HH) Structure	% Change of				Insurance Provided by			
	Male Income	Female Income	Household Income	Household Consumption	Male Earner	Female Earner	Risk Free Bond	Total Insurance
<i>A. Permanent Shock</i>								
(i) 1-earner HH with exogenous labor	1	–	1	0.648	0	–	35.2%	35.2%
(ii) 1-earner HH with endogenous labor	0.740	–	0.740	0.390	26.0%	–	35.0%	61.0%
(iii) 2-earner HH with endogenous male and exogenous female labor	0.899	0	0.567	0.320	10.1%	33.2%	24.7%	68.0%
(iv) 2-earner HH with endogenous labor of both earners	1.067	–0.733	0.405	0.276	–6.7%	66.2%	12.9%	72.4%
<i>B. Transitory Shock</i>								
(i) 1-earner HH with exogenous labor	1	–	1	0.030	0	–	97.0%	97.0%
(ii) 1-earner HH with endogenous labor	2.006	–	2.006	–0.192	–100.6%	–	219.8%	119.2%
(iii) 2-earner HH with endogenous male and exogenous female labor	1.929	0	1.216	–0.111	–92.9%	71.3%	132.7%	111.1%
(iv) 2-earner HH with endogenous labor of both earners	1.946	0.146	1.284	–0.128	–94.6%	66.2%	141.2%	112.8%

Note: The results are based on sample averages of transmission coefficients from different economies with the same parameters as the ZBC economy with non-separable preference but different household structures.

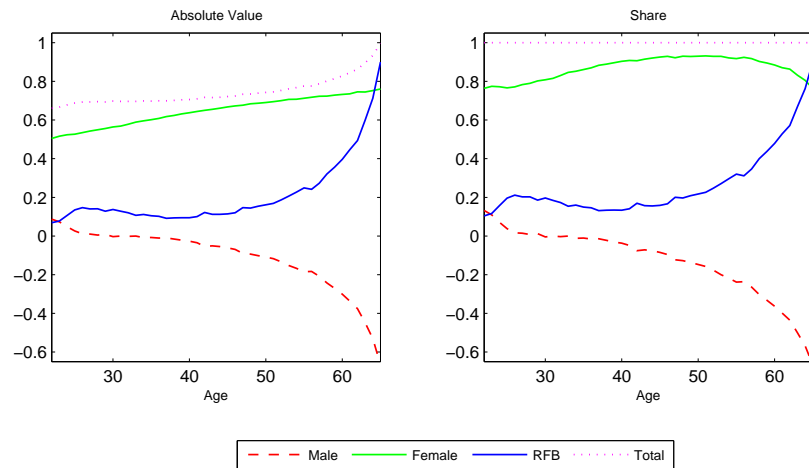


Figure 1.9: Age Profiles of Consumption Insurance Decomposition  
(Non-separable Preference)

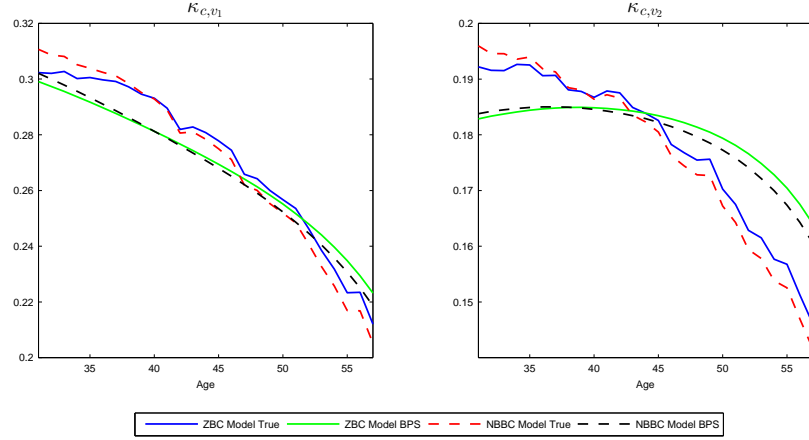


Figure 1.10: Age Profiles of  $\kappa_{c,v_j}$ : Model True vs. Model BPS I (Non-separable Preference)

### 1.A.3 Performance of the BPS Method

## 1.B Calibration Details

### 1.B.1 Wage Profiles

The life-cycle male wage trend is interpolated and extrapolated from Rupert and Zanella (2012) and plotted in Figure 1.11. The scale of it is normalized such that the average male trend wage is 1. The female wage trend is rescaled from the male wage trend to match the ratio of the average female wage to the average male wage, which is 0.685 from the BPS data set.

### 1.B.2 Parameter Values and Moments Matched

The parameter values which are the same for all the economies are reported in Table 1.11. The preference-specific parameter values are reported in Table 1.12 and 1.13 for the additive separable and non-separable preferences, respectively. The moments matched in calibration and their values from data are in Table 1.14. The calibrated Frisch elasticities

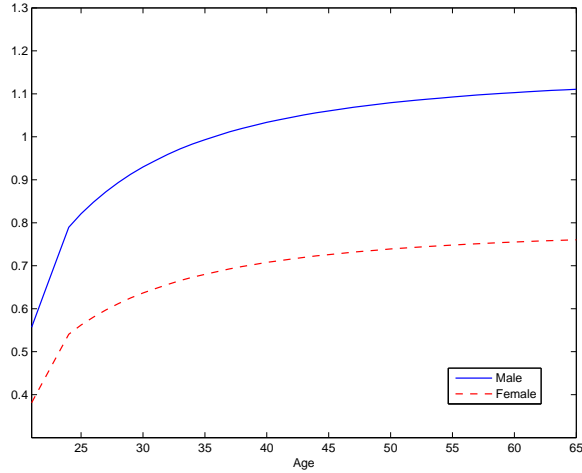


Figure 1.11: Male and Female Wage Trends

and their BPS counterparts for the non-separable preference are in Table 1.15.

Table 1.11: Values of Common Parameters

Parameter	Governing	Value
$r$	interest rate	0.02
<i>Transitory Wage Shocks</i>		
$\sigma_{u_1}^2$	variance of male shocks	0.0297
$\sigma_{u_2}^2$	variance of female shocks	0.0125
$\sigma_{u_1, u_2}$	covariance of male and female shocks	0.0054
<i>Permanent Wage Shocks</i>		
$\sigma_{v_1}^2$	variance of male shocks	0.0294
$\sigma_{v_2}^2$	variance of female shocks	0.0391
$\sigma_{v_1, v_2}$	covariance of male and female shocks	0.0028

## 1.C Frisch Elasticities for the Non-separable Preference

In this section, we derive the formulas of the Frisch elasticities for the non-separable preference. They are generally functions of the household allocations, i.e., they are not deep parameters. So we use the sample averages of them as the approximated true values of the Frisch elasticities, i.e. the “Model True” results.

Table 1.12: Parameter Values for the Additive Separable Preference

Parameter	Governing	ZBC	NBBC
$\delta$	discount rate of utility	0.0119	0.0100
$\psi_1$	disutility of male labor supply	0.394	0.414
$\psi_2$	disutility of female labor supply	0.539	0.565
$b$	retirement benefits	1.155	1.140
$\sigma$	inverse of consumption Frisch elasticity	0.613	0.613
$\eta_1$	male labor supply Frisch elasticity	0.490	0.490
$\eta_2$	female labor supply Frisch elasticity	0.849	0.849
$f$	fixed utility cost of female labor participation	0.023	0.025

Table 1.13: Parameter Values for the Non-separable Preference

Parameter	Governing	ZBC	NBBC
$\delta$	discount rate of utility	0.0162	0.0129
$\alpha$	weight of consumption	0.74	0.70
$\xi$	weight of male labor supply	0.42	0.42
$b$	retirement benefits	0.945	0.921
$\sigma$	consumption Frisch elasticity	2.420	2.432
$\gamma$	substitution between consumption and leisure	-2.70	-2.70
$\theta$	substitution between male and female labor supply	2.25	2.24
$\Psi$	consumption level after retirement	0.68	0.69
$f$	fixed utility cost of female labor participation	0.0083	0.0087

The utility function for the non-separable preference is

$$u(C, H_1, H_2) = \frac{\{\alpha C^\gamma + (1 - \alpha)[\xi H_1^\theta + (1 - \xi)H_2^\theta]^{-\frac{\gamma}{\theta}}\}^{\frac{1-\sigma}{\gamma}} - 1}{1 - \sigma}$$

The intertemporal budget constraint is

$$PC + Pa' = P(1 + r)a + W_1H_1 + W_2H_2$$

where  $P$ ,  $W_1$ , and  $W_2$  are the price of the consumption good and the wages for male and female earners. From the recursive formulation of households' problem, the first order

Table 1.14: Values of Matched Moments

Moment Matched	Target Value
asset/income ratio	2.8251
average male labor supply	1
average female labor supply	0.7711
average benefits/average income ratio	0.4250
average female nonparticipation rate	21%

Table 1.15: Calibrated Frisch Elasticities for the Non-separable Preference

	Data BPS	ZBC	NBBC
		Model True	Model True
$\eta_{c,p}$	0.418(0.095)	0.373	0.369
$\eta_{c,w_1}$	-0.144(0.069)	-0.148	-0.150
$\eta_{c,w_2}$	-0.062(0.077)	-0.086	-0.088
$\eta_{h_1,p}$	0.082(0.040)	0.223	0.215
$\eta_{h_1,w_1}$	0.531(0.120)	0.988	0.996
$\eta_{h_1,w_2}$	0.099(0.054)	0.110	0.110
$\eta_{h_2,p}$	0.073(0.088)	0.223	0.215
$\eta_{h_2,w_1}$	0.201(0.108)	0.188	0.189
$\eta_{h_2,w_2}$	0.903(0.223)	0.910	0.917

conditions are

$$u_C = \Delta^{\frac{1-\sigma}{\gamma}-1} \alpha C^{\gamma-1} = \lambda P$$

$$u_{H_1} = -\Delta^{\frac{1-\sigma}{\gamma}-1} (1-\alpha) \Gamma^{-\frac{\gamma}{\theta}-1} \xi H_1^{\theta-1} = -\lambda W_1$$

$$u_{H_2} = -\Delta^{\frac{1-\sigma}{\gamma}-1} (1-\alpha) \Gamma^{-\frac{\gamma}{\theta}-1} (1-\xi) H_2^{\theta-1} = -\lambda W_2$$

where  $\lambda$  is the lagrangian multiplier on the budget constraint, and  $\Delta \equiv \alpha C^\gamma + (1-\alpha)[\xi H_1^\theta + (1-\xi)H_2^\theta]^{-\frac{\gamma}{\theta}}$  and  $\Gamma \equiv \xi H_1^\theta + (1-\xi)H_2^\theta$ . Taking log difference for both sides of these equations, we can get

$$\begin{aligned} & \left(\frac{1-\sigma}{\gamma} - 1\right)d \ln \Delta + (\gamma - 1)d \ln C = d \ln \lambda + d \ln P \\ & \left(\frac{1-\sigma}{\gamma} - 1\right)d \ln \Delta + \left(-\frac{\gamma}{\theta} - 1\right)d \ln \Gamma + (\theta - 1)d \ln H_1 = d \ln \lambda + d \ln W_1 \\ & \left(\frac{1-\sigma}{\gamma} - 1\right)d \ln \Delta + \left(-\frac{\gamma}{\theta} - 1\right)d \ln \Gamma + (\theta - 1)d \ln H_2 = d \ln \lambda + d \ln W_2 \end{aligned}$$

and

$$\begin{aligned} d \ln \Gamma &= \theta B \ln H_1 + \theta(1 - B)d \ln H_2 \\ d \ln \Delta &= \gamma A d \ln C - \frac{\gamma}{\theta}(1 - A)d \ln \Gamma \\ &= \gamma A d \ln C - \gamma(1 - A)B d \ln H_1 - \gamma(1 - A)(1 - B)d \ln H_2 \end{aligned}$$

where  $A \equiv \frac{\alpha C^\gamma}{\Delta}$  and  $B \equiv \frac{\xi H_1^\theta}{\Gamma}$ . Substitute  $d \ln \Delta$  and  $d \ln \Gamma$  with the formulas above, the system of equations becomes

$$G \times \begin{bmatrix} d \ln C \\ d \ln H_1 \\ d \ln H_2 \end{bmatrix} = \begin{bmatrix} d \ln \lambda + d \ln P \\ d \ln \lambda + d \ln W_1 \\ d \ln \lambda + d \ln W_2 \end{bmatrix}$$

where

$$G = \begin{bmatrix} (\gamma - 1)(1 - A) - \sigma A & (\gamma - 1 + \sigma)(1 - A)B & (\gamma - 1 + \sigma)(1 - A)(1 - B) \\ (1 - \gamma - \sigma)A & [(\gamma - 1 + \sigma)(1 - A) - (\gamma + \theta)]B + (\theta - 1) & [(\gamma - 1 + \sigma)(1 - A) - (\gamma + \theta)](1 - B) \\ (1 - \gamma - \sigma)A & [(\gamma - 1 + \sigma)(1 - A) - (\gamma + \theta)]B & [(\gamma - 1 + \sigma)(1 - A) - (\gamma + \theta)](1 - B) + (\theta - 1) \end{bmatrix}$$



By the definition of Frisch elasticities, we have

$$G^{-1} = \begin{bmatrix} -\eta_{c,p} & \eta_{c,w_1} & \eta_{c,w_2} \\ \eta_{h_1,p} & \eta_{h_1,w_1} & \eta_{h_1,w_2} \\ \eta_{h_2,p} & \eta_{h_2,w_1} & \eta_{h_2,w_2} \end{bmatrix}.$$

Note that because the values of  $A$  and  $B$  depend on the allocations chosen by households,  $G$  and the Frisch elasticities all depends on the allocations and are not deep parameters.

If we want the Frisch elasticities to be deep parameters, we must have  $A$  and  $B$  to be constant. From the FOC's, this requires

$$\begin{aligned} \Delta^{\frac{1-\sigma}{\gamma}} A &= \lambda PC \\ \Delta^{\frac{1-\sigma}{\gamma}} (1-A)B &= \lambda W_1 H_1 \\ \Delta^{\frac{1-\sigma}{\gamma}} (1-A)(1-B) &= \lambda W_2 H_2 \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} \frac{A}{(1-A)B} &= \frac{PC}{W_1 H_1} = \text{Constant} \\ \frac{A}{(1-A)(1-B)} &= \frac{PC}{W_2 H_2} = \text{Constant} \end{aligned}$$

This implies the utility function needs to take the Cobb-Douglas form, i.e.,  $\gamma = 0$ , and  $\theta = 0$ . In that case, the utility function becomes

$$U(C, H_1, H_2) = \frac{[C^\alpha (H_1^\xi H_2^{1-\xi})^{-(1-\alpha)}]^{1-\sigma} - 1}{1-\sigma}$$

Follow the same method, we can derive that

$$G = \begin{bmatrix} \alpha - 1 - \alpha\sigma & (1 - \sigma)(\alpha - 1)\xi & (1 - \sigma)(\alpha - 1)(1 - \xi) \\ (1 - \sigma)\alpha & -\sigma(\alpha - 1)\xi + (\alpha - 2)\xi + (\xi - 1) & -\sigma(\alpha - 1)(1 - \xi) + (\alpha - 2)(1 - \xi) + (1 - \xi) \\ (1 - \sigma)\alpha & -\sigma(\alpha - 1)\xi + (\alpha - 2)\xi + \xi & -\sigma(\alpha - 1)(1 - \xi) + (\alpha - 2)(1 - \xi) - \xi \end{bmatrix}$$

and the Frisch elasticities matrix is just  $G^{-1}$ . However, the Cobb-Douglas form is not a good choice because it implies that the ratios between male, female labor income and consumption expenditures are all constants independent of the prices of them, which is counterfactual.

## 1.D The BPS Method in This Paper

### 1.D.1 Formulas of the Transmission Coefficients

We follow closely the approximation method proposed by Blundell, Pistaferri, and Saporta-Eksten (2014) (BPS), and try to use the notations consistent with the original paper. However, we do make one slight modification to their formulas. In particular, we add the channel of social insurance explicitly. We define the human wealth as the sum of the discounted labor income and the discounted retirement benefits, and the share of the retirement benefits in the total discounted human wealth is represented by  $q_t$ . By doing so, the true value of the external insurance coefficient  $\beta$  in the model should be zero. In the BPS paper, they did not model the social security benefits explicitly, and the effect of them is essentially captured by the external insurance coefficient  $\beta$ .

As in the BPS paper, the wage is determined by

$$\ln W_{i,j,t} = Z_t^{W_j'} \beta^{W_j} + F_{i,j,t} + u_{i,j,t}$$

where  $Z_t^{W_j}$  are a group of observable characteristics affecting wages and known to the

households such as age; and

$$F_{i,j,t} = F_{i,j,t-1} + v_{i,j,t}.$$

This implies

$$\Delta \ln W_{i,j,t} - \Delta Z_t^{W_j'} \beta^{W_j} = \Delta u_{i,j,t} + v_{i,j,t}.$$

Define  $\Delta w_{i,j,t}$  as the unexpected growth of wage which is not explained by the observables, i.e.,

$$\Delta w_{i,j,t} = \Delta u_{i,j,t} + v_{i,j,t}.$$

Log linearize the first order conditions of the household's problem, and apply the definition of the Frisch elasticities, we have

$$\begin{bmatrix} d \ln C \\ d \ln H_1 \\ d \ln H_2 \end{bmatrix} = \begin{bmatrix} -\eta_{c,p} & \eta_{c,w_1} & \eta_{c,w_2} \\ \eta_{h_1,p} & \eta_{h_1,w_1} & \eta_{h_1,w_2} \\ \eta_{h_2,p} & \eta_{h_2,w_1} & \eta_{h_2,w_2} \end{bmatrix} \times \begin{bmatrix} d \ln \lambda + d \ln P \\ d \ln \lambda + d \ln W_1 \\ d \ln \lambda + d \ln W_2 \end{bmatrix}.$$

The BPS paper derived that

$$\Delta \ln \lambda_t = \omega_t + \varepsilon_t$$

where  $\omega_t$  only depends on age, hence will be absorbed by the age dummies in regressions.

Then we have

$$\begin{aligned} \Delta c_t &= (-\eta_{c,p} + \eta_{c,w_1} + \eta_{c,w_2})\varepsilon_t + \eta_{c,w_1}(\Delta u_{1,t} + v_{1,t}) + \eta_{c,w_2}(\Delta u_{2,t} + v_{2,t}) \\ \Delta h_{1,t} &= (\eta_{h_1,p} + \eta_{h_1,w_1} + \eta_{h_1,w_2})\varepsilon_t + \eta_{h_1,w_1}(\Delta u_{1,t} + v_{1,t}) + \eta_{h_1,w_2}(\Delta u_{2,t} + v_{2,t}) \\ \Delta h_{2,t} &= (\eta_{h_2,p} + \eta_{h_2,w_1} + \eta_{h_2,w_2})\varepsilon_t + \eta_{h_2,w_1}(\Delta u_{1,t} + v_{1,t}) + \eta_{h_2,w_2}(\Delta u_{2,t} + v_{2,t}) \end{aligned}$$

where  $\Delta c_t$  and  $\Delta h_{j,t}$  represent the unexpected growth of consumption and labor supplies which are not explained by the observables. Transform the labor supplies into labor in-

comes, we have

$$\begin{aligned}
\Delta c_t &= (-\eta_{c,p} + \eta_{c,w_1} + \eta_{c,w_2})\varepsilon_t + \eta_{c,w_1}(\Delta u_{1,t} + v_{1,t}) + \eta_{c,w_2}(\Delta u_{2,t} + v_{2,t}) \\
\Delta y_{1,t} &= (\eta_{h_1,p} + \eta_{h_1,w_1} + \eta_{h_1,w_2})\varepsilon_t + (1 + \eta_{h_1,w_1})(\Delta u_{1,t} + v_{1,t}) + \eta_{h_1,w_2}(\Delta u_{2,t} + v_{2,t}) \\
\Delta y_{2,t} &= (\eta_{h_2,p} + \eta_{h_2,w_1} + \eta_{h_2,w_2})\varepsilon_t + \eta_{h_2,w_1}(\Delta u_{1,t} + v_{1,t}) + (1 + \eta_{h_2,w_2})(\Delta u_{2,t} + v_{2,t})
\end{aligned} \tag{1.D.1}$$

where  $\Delta y_{j,t}$  is the unexpected income growth of earner  $j$  at age  $t$ .

From the log linearization of the intertemporal budget constraint, assuming the transitory shocks have negligible wealth effects, the BPS paper derived that

$$\begin{aligned}
&(-\eta_{c,p} + \eta_{c,w_1} + \eta_{c,w_2})\varepsilon_t + \eta_{c,w_1}v_{1,t} + \eta_{c,w_2}v_{2,t} \\
&= (1 - \pi_t)q_t \sum_{j=1}^2 s_j [(1 + \eta_{h_j,w_j})v_{j,t} + \eta_{h_j,w_{-j}}v_{-j,t} + (\eta_{h_j,p} + \eta_{h_j,w_j} + \eta_{h_j,w_{-j}})\varepsilon_t]
\end{aligned}$$

where  $\pi_t$  is approximately the share of asset in the total discounted wealth for the household at age  $t$ ;  $s_{j,t}$  is approximately the share of earner  $j$ 's discounted labor income in the total discounted labor income of the household;  $1 - q_t$  is the share of the retirement benefits in the total discounted wealth of the household at age  $t$ . From this, we can get the formula for the  $\varepsilon_t$ ,

$$\begin{aligned}
\varepsilon_t &= \frac{\eta_{c,w_1} - (1 - \pi_t)q_t[s_1 + \overline{\eta_{h,w_1}}]}{\eta_{c,p} - \eta_{c,w_1} - \eta_{c,w_2} + (1 - \pi_t)q_t(\overline{\eta_{h,p}} + \overline{\eta_{h,w_1}} + \overline{\eta_{h,w_2}})}v_{1,t} \\
&+ \frac{\eta_{c,w_2} - (1 - \pi_t)q_t[s_2 + \overline{\eta_{h,w_2}}]}{\eta_{c,p} - \eta_{c,w_1} - \eta_{c,w_2} + (1 - \pi_t)q_t(\overline{\eta_{h,p}} + \overline{\eta_{h,w_1}} + \overline{\eta_{h,w_2}})}v_{2,t} \\
&\equiv \kappa_{\varepsilon,v_1}v_{1,t} + \kappa_{\varepsilon,v_2}v_{2,t}
\end{aligned}$$

where  $\overline{\eta_{h,p}} \equiv \sum_{j=1}^2 s_j \eta_{h_j,p}$ ,  $\overline{\eta_{h,w_1}} \equiv \sum_{j=1}^2 s_j \eta_{h_j,w_1}$  and  $\overline{\eta_{h,w_2}} \equiv \sum_{j=1}^2 s_j \eta_{h_j,w_2}$ .

Substitute the result for the  $\varepsilon_t$  into Equations (1.D.1), and we have the formulas for the transmission coefficients:

$$\begin{bmatrix} \Delta C_t \\ \Delta y_{1,t} \\ \Delta y_{2,t} \end{bmatrix} = \begin{bmatrix} \kappa_{c,u_1} & \kappa_{c,u_2} & \kappa_{c,v_1} & \kappa_{c,v_2} \\ \kappa_{y_1,u_1} & \kappa_{y_1,u_2} & \kappa_{y_1,v_1} & \kappa_{y_1,v_2} \\ \kappa_{y_2,u_1} & \kappa_{y_2,u_2} & \kappa_{y_2,v_1} & \kappa_{y_2,v_2} \end{bmatrix} \begin{bmatrix} \Delta u_{1,t} \\ \Delta u_{2,t} \\ v_{1,t} \\ v_{2,t} \end{bmatrix}$$

where

$$\kappa_{c,u_j} = \eta_{c,w_j}$$

$$\kappa_{c,v_j} = (-\eta_{c,p} + \eta_{c,w_1} + \eta_{c,w_2})\kappa_{\varepsilon,v_j} + \eta_{c,w_j}$$

$$\kappa_{y_j,u_j} = 1 + \eta_{h_j,w_j}$$

$$\kappa_{y_j,u_{-j}} = \eta_{h_j,w_{-j}}$$

$$\kappa_{y_j,v_j} = (\eta_{h_j,p} + \eta_{h_j,w_j} + \eta_{h_j,w_{-j}})\kappa_{\varepsilon,v_j} + (1 + \eta_{h_j,w_j})$$

$$\kappa_{y_j,v_{-j}} = (\eta_{h_j,p} + \eta_{h_j,w_j} + \eta_{h_j,w_{-j}})\kappa_{\varepsilon,v_{-j}} + \eta_{h_j,w_{-j}}$$

and

$$\kappa_{\varepsilon,v_j} = \frac{\eta_{c,w_j} - (1 - \pi_t)q_t[s_j + \overline{\eta_{h,w_j}}]}{\eta_{c,p} - \eta_{c,w_1} - \eta_{c,w_2} + (1 - \pi_t)q_t(\overline{\eta_{h,p}} + \overline{\eta_{h,w_1}} + \overline{\eta_{h,w_2}})}$$

The formulas when the additive separable preference assumption is imposed can be obtained by assuming the values of all the cross Frisch elasticities to be zero.

## 1.D.2 Estimation

The estimation method in this paper follows the empirical strategy and identification Appendix in the original BPS paper. To apply the method, we first need the data on the

unexpected wage growth  $\Delta w_{j,t}$ , unexpected consumption growth  $\Delta c_t$ , and unexpected labor income growth  $\Delta y_{j,t}$  at household level. These can be obtained by regressing the log differences of the corresponding variables onto observable characteristics and constructing the residuals, i.e.,

$$\Delta \log X_t = Z\beta + \Delta x_t$$

where  $Z$  are the observable characteristics. For the simulated data, because all the households are ex ante identical,  $Z$  only includes a group of age dummies.

### Wage Covariances

From the wage process, we know

$$\Delta w_{j,t} = \Delta u_{j,t} + v_{j,t}.$$

Hence the variances and covariances of the wage shocks can be estimated by

$$\begin{aligned}\sigma_{u_1}^2 &= -E[\Delta w_{1,t}\Delta w_{1,t+1}] \\ \sigma_{u_2}^2 &= -E[\Delta w_{2,t}\Delta w_{2,t+1}] \\ \sigma_{u_1,u_2} &= -E[\Delta w_{2,t}\Delta w_{1,t+1}] \\ \sigma_{v_1}^2 &= E[\Delta w_{1,t}(\Delta w_{1,t+1} + \Delta w_{1,t} + \Delta w_{1,t-1})] \\ \sigma_{v_2}^2 &= E[\Delta w_{2,t}(\Delta w_{2,t+1} + \Delta w_{2,t} + \Delta w_{2,t-1})] \\ \sigma_{v_1,v_2} &= E[\Delta w_{1,t}(\Delta w_{2,t+1} + \Delta w_{2,t} + \Delta w_{2,t-1})]\end{aligned}$$

## Frisch Elasticities excluding $\eta_{c,p}$ and $\beta$

Calculate the following moments from the data:

$$m_1 = E[\Delta w_{1,t} \Delta y_{1,t+1}] = -(1 + \eta_{h_1, w_1}) \sigma_{u_1}^2 - \eta_{h_1, w_2} \sigma_{u_1, u_2} = (1 + \eta_{h_1, w_1}) m_3 + \eta_{h_1, w_2} m_5$$

$$m_2 = E[\Delta w_{2,t} \Delta y_{1,t+1}] = -(1 + \eta_{h_1, w_1}) \sigma_{u_1, u_2} - \eta_{h_1, w_2} \sigma_{u_2}^2 = (1 + \eta_{h_1, w_1}) m_5 + \eta_{h_1, w_2} m_4$$

$$m_3 = E[\Delta w_{1,t} \Delta w_{1,t+1}] = -\sigma_{u_1}^2$$

$$m_4 = E[\Delta w_{2,t} \Delta w_{2,t+1}] = -\sigma_{u_2}^2$$

$$m_5 = E[\Delta w_{2,t} \Delta w_{1,t+1}] = -\sigma_{u_1, u_2}$$

$$m_6 = E[\Delta w_{1,t} \Delta c_{t+1}] = -\eta_{c, w_1} \sigma_{u_1}^2 - \eta_{c, w_2} \sigma_{u_1, u_2} = \eta_{c, w_1} m_3 + \eta_{c, w_2} m_5$$

$$m_7 = E[\Delta w_{2,t} \Delta c_{t+1}] = -\eta_{c, w_1} \sigma_{u_1, u_2} - \eta_{c, w_2} \sigma_{u_2}^2 = \eta_{c, w_1} m_5 + \eta_{c, w_2} m_4$$

$$m'_1 = E[\Delta w_{2,t} \Delta y_{2,t+1}] = -(1 + \eta_{h_2, w_2}) \sigma_{u_2}^2 - \eta_{h_2, w_1} \sigma_{u_1, u_2} = (1 + \eta_{h_2, w_2}) m_4 + \eta_{h_2, w_1} m_5$$

$$m'_2 = E[\Delta w_{1,t} \Delta y_{2,t+1}] = -(1 + \eta_{h_2, w_2}) \sigma_{u_1, u_2} - \eta_{h_2, w_1} \sigma_{u_1}^2 = (1 + \eta_{h_2, w_2}) m_5 + \eta_{h_2, w_1} m_3$$

Then we have

$$\eta_{h_1, w_1} = \frac{m_1 m_4 - m_2 m_5}{m_3 m_4 - m_5^2} - 1$$

$$\eta_{h_1, w_2} = \frac{m_2 m_3 - m_1 m_5}{m_3 m_4 - m_5^2}$$

$$\eta_{h_2, w_1} = \frac{m'_1 m_4 - m'_2 m_5}{m_3 m_4 - m_5^2}$$

$$\eta_{h_2, w_2} = \frac{m'_1 m_3 - m'_2 m_5}{m_3 m_4 - m_5^2} - 1$$

$$\eta_{c, w_1} = \frac{m_6 m_4 - m_7 m_5}{m_3 m_4 - m_5^2}$$

$$\eta_{c, w_2} = \frac{m_7 m_3 - m_6 m_5}{m_3 m_4 - m_5^2}$$

By imposing symmetry (details in the original BPS paper Section 4.2.2), we have

$$\eta_{h_j,p} = -\eta_{c,w_j} \frac{pc}{w_j h_j}, j = 1, 2.$$

### Smoothing Parameters

The smoothing parameters  $\pi_t$ ,  $s_{j,t}$  and  $q_t$  are calculated directly from the data. The labor wealth of earner  $j$  at age  $t$  is calculated as

$$Labor\ Wealth_{j,t} = Y_{j,t} + E_t \sum_{k=1}^{R-t} \frac{Y_{j,t+k}}{(1+r)^k}.$$

Note that the expected future incomes should technically depend on the current states of the households. However, in practice, it is hard to calculate the conditional expectations, so following the original BPS paper, the unconditional income levels are used. Then  $s_{j,t}$  is simply

$$s_{j,t} = \frac{Labor\ Wealth_{j,t}}{\sum_{j=1}^2 Labor\ Wealth_{j,t}}.$$

The retirement wealth, which we define as the sum of the discounted retirement benefits, for a household at age  $t$  is

$$Retirement\ Wealth_t = \frac{1}{(1+r)^{R-t}} \sum_{k=1}^{T-R} \frac{b}{(1+r)^k}.$$

We define the human wealth of a household at age  $t$  as

$$Human\ Wealth_t = Retirement\ Wealth_t + \sum_{j=1}^2 Labor\ Wealth_{j,t}.$$

So  $q_t$  is

$$q_t = \frac{Retirement\ Wealth_t}{Human\ Wealth_t},$$



and  $\pi_t$  is

$$\pi_t = \frac{Assets_t}{Assets_t + Human\ Wealth_t}$$

$\eta_{c,p}$  and  $\beta$

Conditional on the estimates from previous steps, we use five moment conditions to jointly estimate  $\eta_{c,p}$  (and  $\beta$  if needed):  $E(\Delta c_t^2)$ ,  $E(\Delta y_{1,t}^2)$ ,  $E(\Delta c_t \Delta y_{1,t})$ ,  $E(\Delta c_t \Delta w_{1,t})$ , and  $E(\Delta y_{1,t} \Delta w_{1,t})$ . We do not use moment conditions related to female to avoid the selection problem due to the extensive margin of female labor supply. Because we have large sample size for the simulated data, there are no efficiency costs from using fewer moment conditions. We use the iterated GMM method in which the weighting matrix is initialized as the identity matrix and then updated optimally based on the estimation results from last step until the estimation results converge. The formulas for these moment conditions are derived based on the BPS formulas for  $\Delta c_t$ ,  $\Delta y_{1,t}$  and  $\Delta w_{1,t}$ . For example,

$$\begin{aligned} E(\Delta c_t^2) &= E[(\kappa_{c,u1} \Delta u_{1,t} + \kappa_{c,u2} \Delta u_{2,t} + \kappa_{c,v1} v_{1,t} + \kappa_{c,v2} v_{2,t})^2] \\ &= E[\kappa_{c,u1}^2 (2\sigma_{u1}^2) + \kappa_{c,u2}^2 (2\sigma_{u2}^2) + 2(\kappa_{c,u1} \kappa_{c,u2}) (2\sigma_{u1u2}) \\ &\quad + \kappa_{c,v1}^2 (\sigma_{v1}^2) + \kappa_{c,v2}^2 E(\sigma_{v2}^2) + 2(\kappa_{c,v1} \kappa_{c,v2}) (\sigma_{v1v2})] \\ &= p \lim \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T \{ \kappa_{c,u1}^2 (i, t) (2\sigma_{u1}^2) + \kappa_{c,u2}^2 (i, t) (2\sigma_{u2}^2) + 2[\kappa_{c,u1} (i, t) \kappa_{c,u2} (i, t)] (2\sigma_{u1u2}) \\ &\quad + \kappa_{c,v1}^2 (i, t) (\sigma_{v1}^2) + \kappa_{c,v2}^2 (i, t) E(\sigma_{v2}^2) + 2[\kappa_{c,v1} (i, t) \kappa_{c,v2} (i, t)] (\sigma_{v1v2}) \}. \end{aligned}$$

The results for the other moment conditions can be derived in the similar way.

### Transmission Coefficients

Collecting the estimation results from previous steps, the transmission coefficients for each household at each age are calculated using the formulas derived in Appendix 1.D.1.

The reported transmission coefficients are the sample averages of them.

## 1.E Computation Method

The household's problem is solved backwards using the endogenous grid method proposed by Carroll (2006) and the policy function iteration through the Euler equations. When the extensive margin of female labor supply is allowed, for each iteration and each state of the household, the model is solved twice for two alternative cases, i.e., the current period female labor supply is strictly positive or zero. The final optimal policy is obtained by comparing the maximum discounted utility attained in the two cases.

The grids for asset have 100 grid points, and the distance between two adjacent points is increasing as the value of asset increases such that the grid points are denser around the low value area. The range of asset grids are adjusted at different ages to have a better coverage of the more relevant areas. The joint process of two earners' permanent wage components is approximated by a Markov process with age-dependent sets of states, and each state corresponds to a possible realization of the pair of two permanent components. The number of grid points at each age is constant, but the values of states are adjusted to match the unconditional dispersion of the joint distribution. The grid points and the associated age-dependent transition matrices are generated following the spirit of Tauchen (1986), and try to mimic the joint unit root process. The grids for the two permanent components have 11 points on each dimension, so there are in total 121 grid points at each age. The discretization of the transitory components is similar and simpler. Because the transitory components are i.i.d across different ages, the grids and transition matrices are the same at different ages. The grid for the transitory components has 5 points on each dimension, so there are in total 25 grid points.

Below we provide the derivations of the Euler equations used in computation when the

female labor supply is strictly positive. When the female labor supply is zero, one only needs to impose this restriction properly in the first order conditions, and follows the same steps.

### 1.E.1 Additive Separable Preference

For a working household, the first order conditions are

$$C^{-\sigma} = \lambda$$

$$-\psi_1 H_1^{\eta_1 - 1} + \lambda e^{h_{1,t} + F_1 + u_1} \leq 0 \text{ with equality when } H_1 > 0$$

$$-\psi_2 H_2^{\eta_2 - 1} + \lambda e^{h_{2,t} + F_2 + u_2} \leq 0 \text{ with equality when } H_2 > 0$$

and the Euler equation is

$$C^{-\sigma} \geq \frac{1+r}{1+\delta} \sum_{(F'_1, F'_2)} \pi(F'_1, F'_2 | F_1, F_2) \sum_{(u'_1, u'_2)} \pi(u'_1, u'_2) C'^{-\sigma} \text{ with equality when } A' > \underline{A}_{t+1}.$$

For a retired household, let the period utility function be

$$u^R(C) = \frac{C^{1-\sigma}}{1-\sigma}$$

then the Euler equation is

$$C^{-\sigma} \geq \frac{1+r}{1+\delta} C'^{-\sigma} \text{ with equality when } A' > \underline{A}_{t+1}.$$

Given  $c(A, F'_1, F'_2, u'_1, u'_2, t+1)$ , there is a closed form solution to the current consumption level when  $A' > \underline{A}_{t+1}$ . For the current  $A$  values such that  $A' = \underline{A}_{t+1}$ , the current consumption level is solved from the budget constraint. Once  $C$  is known,  $H_1$  and  $H_2$  are

just closed form functions of  $C$ .

## 1.E.2 Non-separable Preference

For a working household, the first order conditions are

$$\Delta^{\frac{1-\sigma}{\gamma}-1} \alpha C^{\gamma-1} = \lambda P$$

$$\Delta^{\frac{1-\sigma}{\gamma}-1} (1-\alpha) \Gamma^{-\frac{\gamma}{\theta}-1} \xi H_1^{\theta-1} \geq \lambda W_1(t, F_1, F_2, u_1, u_2) \text{ with equality when } H_1 > 0$$

$$\Delta^{\frac{1-\sigma}{\gamma}-1} (1-\alpha) \Gamma^{-\frac{\gamma}{\theta}-1} (1-\xi) H_2^{\theta-1} \geq \lambda W_2(t, F_1, F_2, u_1, u_2) \text{ with equality when } H_2 > 0$$

where  $\Delta \equiv \alpha C^\gamma + (1-\alpha)[\xi H_1^\theta + (1-\xi)H_2^\theta]^{-\frac{\gamma}{\theta}}$  and  $\Gamma \equiv \xi H_1^\theta + (1-\xi)H_2^\theta$ . And the Euler equation for a working household is

$$\begin{aligned} \Delta^{\frac{1-\sigma}{\gamma}-1} c(A, F_1, F_2, u_1, u_2, t)^{\gamma-1} &\geq \\ \frac{1+r}{1+\delta} \sum_{(F'_1, F'_2)} \pi(F'_1, F'_2 | F_1, F_2) \sum_{(u'_1, u'_2)} \pi(u'_1, u'_2) \Delta'^{\frac{1-\sigma}{\gamma}-1} c(A', F'_1, F'_2, u'_1, u'_2, t+1)^{\gamma-1} & \\ &\text{with equality when } A' > \underline{A}_{t+1} \end{aligned}$$

The Euler equation for a retired household is

$$c(A, t)^{-\sigma_R} \geq \frac{1+r}{1+\delta} c(A', t+1)^{-\sigma_R} \text{ with equality when } A' > \underline{A}_{t+1}$$

For an age  $R$  household, the Euler equation is

$$\begin{aligned} \alpha \Delta^{\frac{1-\sigma}{\gamma}-1} c(A, F_1, F_2, u_1, u_2, R)^{\gamma-1} &\geq \frac{1+r}{1+\delta} \Psi c(A', R+1)^{-\sigma_R} \\ &\text{with equality when } A' > \underline{A}_{R+1} \end{aligned}$$

From the FOC's and the utility function, first we know  $H_1$  is always positive for strictly positive wages. So if female labor supply is also strictly positive, we have

$$\frac{\xi H_1^{\theta-1}}{(1-\xi)H_2^{\theta-1}} = \frac{W_1}{W_2}$$

$$\frac{\alpha C^{\gamma-1}}{(1-\alpha)\Gamma^{-\frac{\gamma}{\theta}-1}(1-\xi)H_2^{\theta-1}} = \frac{P}{W_2}$$

After some algebra, we have

$$H_1$$

$$= \{C^{1-\gamma} P \frac{(1-\alpha)}{\alpha} \{\xi[(1-\xi)W_1]^{\frac{\theta}{\theta-1}} + (1-\xi)(\xi W_2)^{\frac{\theta}{\theta-1}}\}^{-\frac{\gamma}{\theta}-1} \frac{\xi}{W_1} [(1-\xi)W_1]^{\frac{\gamma+\theta}{\theta-1}}\}^{\frac{1}{\gamma+1}}$$

$$\equiv \phi_1 C^{\frac{1-\gamma}{\gamma+1}}$$

$$H_2$$

$$= \{C^{1-\gamma} P \frac{(1-\alpha)}{\alpha} \{\xi[(1-\xi)W_1]^{\frac{\theta}{\theta-1}} + (1-\xi)(\xi W_2)^{\frac{\theta}{\theta-1}}\}^{-\frac{\gamma}{\theta}-1} \frac{(1-\xi)}{W_2} (\xi W_2)^{\frac{\gamma+\theta}{\theta-1}}\}^{\frac{1}{\gamma+1}}$$

$$\equiv \phi_2 C^{\frac{1-\gamma}{\gamma+1}}$$

Substitute  $H_1$  and  $H_2$  in the marginal utility with the formulas above,

$$MU(C, H_1, H_2) = \alpha \Delta^{\frac{1-\sigma}{\gamma}-1} C^{\gamma-1}$$

$$= \alpha \{\alpha + (1-\alpha)[\xi\phi_1^\theta + (1-\xi)\phi_2^\theta]^{-\frac{\gamma}{\theta}} C^{\frac{-2\gamma}{\gamma+1}}\}^{\frac{1-\sigma-\gamma}{\gamma}} C^{-\sigma}$$

After retirement,

$$MU(C) = \Psi C^{-\sigma}$$

So given the next period policy function  $c'$ , we can calculate the marginal utility in the next period. Then using the Euler equations, we can solve the current  $C$  if the borrowing constraint is not binding. Unfortunately, no closed form solution exist for this kind of util-

ity function except for the case with  $\gamma = 0$  and  $\theta = 0$ . Once current consumption level is known, the labor supplies are just closed form functions of  $C$ . If the borrowing constraint binds, the current consumption can again be solved from the intertemporal budget constraint.

## Chapter 2

# More Unequal Income but Less

# Progressive Taxation: Economics or Politics?<sup>1</sup>

Chunzan Wu

### 2.1 Introduction

Since the 1970s, income inequality in the U.S. has increased sharply.<sup>2</sup> Most of this rising income inequality is due to the more unequal labor income in the upper half of the income distribution (Piketty and Saez (2003)).<sup>3</sup> This is true both at the household and individual level, and for both males and females. The rising income inequality has become

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<sup>2</sup>Appendix 2.A provides more information on the empirical facts mentioned in the introduction.

<sup>3</sup>Hence, this paper focuses on the changing labor income structure and labor income tax policy and abstracts away from capital income taxation.

a primary concern for people in the U.S., and a popular suggestion in terms of economic policy to counter such rising income inequality is to adopt a more progressive income tax policy to reduce the after-tax income inequality. However, the actual income tax policy in the U.S. moved in the opposite direction. Changes in income tax law since the late 1970s resulted in larger tax cuts for high-income households, and the income tax schedule today is less progressive than it used to be in the 1970s.

There are two potential explanations for this less progressive income tax despite the rising income inequality. The first one is that the policy-making system might have become more favorable to the high-income households, which could be due to changes in the political influences of various income groups. The second explanation is that economic changes since the 1970s might have increased the cost of progressive income taxation and hence require a less progressive optimal income tax to be adopted.

Several economic changes since the 1970s could potentially contribute to a less progressive optimal income tax policy. The first is skill-biased technological change, which has become the most influential explanation for the rising income inequality in the first place (Acemoglu (2002)). On the one hand, the higher income inequality caused by this technological change increased the redistribution benefits of a more progressive income tax. On the other hand, it also increased the value of human capital. If human capital has to be accumulated endogenously with nontrivial costs, the benefits of a less progressive income tax to encourage human capital investment may surpass the benefits of redistribution and therefore require a less progressive income tax to be optimal.

The declining gender income gap due to increased female labor productivity could also help explain this less progressive income tax. In the late 1970s, the female labor income share in the U.S. was only about a quarter of the total labor income, but in the early 2010s, that ratio climbed to about 40%.<sup>4</sup> The greater female labor income share is caused by the

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<sup>4</sup>Based on the data from the Current Population Survey Annual Social and Economic Supplement.



increase in both the female wage rate and the female labor supply relative to the male wage rate and labor supply (Blau and Kahn (2000), O’Neill (2003)). Empirical studies have found that the female labor supply is more elastic compared with the male labor supply.<sup>5</sup> Hence, as the female labor income takes a larger share of the total household income, the elasticity of household income with respect to marginal income tax increases. Since most married couples file their income taxes jointly, the increased elasticity of household income means the optimal income tax should be less progressive.

A third economic change that might require a less progressive income tax is the aging U.S. population. Life expectancy was 74 in the late 1970s and rose to 79 in the early 2010s. The rising life expectancy increases the age dependency ratio<sup>6</sup> and demands more revenues to be collected through income taxes on the working-age population because: first, it increases the total cost of social security benefits; and second, it increases the demand for government services, as more people are living longer. Previous studies have shown that a more progressive income tax system tends to reduce the government’s ability to extract tax revenues from the economy (Holter, Krueger, and Stepanchuk (2014), Guner, Lopez-Daneri, and Ventura (2015)), and hence, this extra demand for tax revenues could force the policymakers to compromise on redistribution and apply a less progressive income tax policy.

Motivated by the discussions above, the main purpose of this paper is to investigate whether the less progressive income tax policy since the 1970s can be rationalized as an optimal response of tax policy to the changes in economic fundamentals, or whether it is a result of the changing preferences of policymakers over different households. To accomplish this goal, I employ the Ramsey optimal tax policy framework, in which a Ram-

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<sup>5</sup>See Blundell, Pistaferri, and Saporta-Eksten (2012) for a recent empirical estimation of the male and female labor supply elasticities using PSID data.

<sup>6</sup>The age dependency ratio is defined as the ratio of dependents, people younger than 15 or older than 64, to the working-age population, those ages 15 to 64.

sey government chooses the tax policy to maximize a weighted sum of household welfare across different competitive equilibria. Within this framework, the tax policy is determined by: (1) a set of Pareto weights representing the government's preference over different households; and (2) household lifetime utilities summarizing the effects of economic fundamentals. The changes in economic fundamentals since the 1970s are quantified using an economic model with household heterogeneity disciplined by the data in the 1970s and 2010s. The Pareto weights are harder to measure directly but can be inferred from the actual tax policies chosen by inverting the Ramsey problem, i.e., finding the Pareto weights that rationalize the actual tax policy as the solution to the Ramsey problem. By construction, the combination of the change in Pareto weights inferred this way and the economic changes in the model replicates exactly the change of income tax policy in the data. A counterfactual experiment in which economic changes are introduced whereas holding the Pareto weights fixed at the 1970s values identifies the part of income tax change that serves as an optimal response to the underlying economic forces, and the remaining part is then attributed to the change in Pareto weights. I interpret the latter as changes in the political influences of various income groups.

The economic model employed to capture the changes in economic fundamentals since the 1970s and their implications on household behaviors and welfare is a quantitative overlapping generations incomplete-markets life-cycle model with heterogeneous households. To model skill-biased technological change, a Ben-Porath style human capital accumulation technology is introduced as in Guvenen and Kuruscu (2010) and Guvenen, Kuruscu, and Ozkan (2014). The return to human capital investment depends on the heterogeneous learning abilities of earners and increases from the 1970s to 2010s to match the widening gap in the upper half of the labor income distribution. To account explicitly for the changing role of the female labor supply in the economy, each household in the model consists of two earners, a male and a female, and they make joint decisions on household con-

sumption, savings, labor supply, and human capital investment. Female labor productivity increases between the 1970s and 2010s to match the declining gender income gap in the data. Finally, earners face the uninsurable idiosyncratic risk of labor productivity both at labor market entry and over the span of the household life cycle and are subject to tight borrowing constraints. The amount of idiosyncratic risk in the model is calibrated to match the dispersion of labor income among young earners and the earnings dynamics over the life cycle in the data.

The first main finding of my quantitative analysis is that the changes in economic fundamentals since the 1970s alone require a less progressive optimal income tax policy to be adopted and can quantitatively account for 40% of the reduction in progressivity we observe. The progressivity of income tax here is measured by the elasticity of after-tax income with respect to before-tax income.<sup>7</sup> In the late 1970s, this elasticity was about 0.856, and it increased to 0.914 in the early 2010s. The model implies that if the Pareto weights remained the same as they were in the 1970s, the optimal income tax policy in the early 2010s should be with a elasticity of 0.879. Counterfactual experiments show that skill-biased technological change, increased female labor productivity, and the aging U.S. population all contribute to the less progressive optimal income tax and account for 18%, 44%, and 73% of the actual change in progressivity, respectively.<sup>8</sup> However, the effects of these changes are partially offset by the increase of idiosyncratic risk, which increases the insurance benefits of progressive income taxes and accounts for –88% of the actual change in progressivity.

The second main finding is that the Pareto weights, as implied by the actual income tax

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<sup>7</sup>With a flat rate income tax, this elasticity is 1. The smaller is this elasticity, the more progressive is the income tax system.

<sup>8</sup>The contribution of skill-biased technological change may seem small compared with the other two. However, this does not mean that the endogenous human capital accumulation channel is less important. The reason is that skill-biased technological change also causes a significant rise in income inequality at the same time, which would have moved the tax policy in the opposite direction if the endogenous human capital accumulation channel were absent.

policies in the 1970s and 2010s, have changed in two dimensions: (1) the Pareto weights of the high-income households have increased relative to those at the lower end of the income distribution; and (2) the Pareto weights on household private utilities have increased relative to the weight on government services. The first change is most responsible for the remaining 60% of reduction in income tax progressivity, while the second change is most responsible for the significant fall in the overall level of the U.S. income tax since the 1970s.

Finally, since the model ascribes a significant part of observed changes in the income tax policy to changes in political influences, as approximated by the Pareto weights, in the last part of this paper, I provide potential political economy explanations for this phenomenon. Using a stylized probabilistic voting model with political contributions, I show that the lower cost of conveying information to swing voters due to information technology improvements leads to an increased demand for campaign expenditures, as observed in the data. I show that this induces a change in Pareto weights benefiting the high-income households, consistent with the change in income tax policy studied in the first part of this paper. I also show that the rising inequality of voter turnout among different socioeconomic groups may have contributed to such a change in Pareto weights as well.

### **2.1.1 Related Literature**

In terms of the model, this paper is first related to the literature that has studied heterogeneous household models with idiosyncratic risks, as in Huggett (1993) and Aiyagari (1994). This type of models have been widely used in quantitative macroeconomic studies of income, wealth, and consumption inequality and redistributive policies. The model built in this paper is a natural combination of two recent developments of the heterogeneous households life-cycle models. The first development is the adoption of the two-earner household structure, which takes into account not only the role of females in the economy, but also the interaction of behaviors within households. Heathcote, Storesletten, and Violante (2010)

and Guner, Kaygusuz, and Ventura (2012) are recent studies with such feature of the model. The other development is the introduction of endogenous human capital accumulation in quantitative life-cycle models such as Huggett, Ventura, and Yaron (2011) and Guvenen, Kuruscu, and Ozkan (2014) with the Ben-Porath human capital accumulation technology.

In terms of the topic, this paper is in line with the quantitative Ramsey optimal income tax policy literature using heterogeneous agents incomplete-markets life-cycle models, which departs from the previous static optimal income tax studies such as Mirrlees (1971), Diamond (1998), and Saez (2001). Important works in this direction include Conesa and Krueger (2006), Conesa, Kitao, and Krueger (2009), and most recent Badel and Huggett (2014), Kindermann and Krueger (2014), and Guner, Lopez-Daneri, and Ventura (2015) on the income tax policy for the top 1%. Krueger and Ludwig (2013) ask the optimal labor income tax question with endogenous college education decisions, whereas my paper focuses on the human capital accumulation over the working life cycle. Heathcote, Storesletten, and Violante (2014) develop an analytic framework to link the economic fundamentals to the optimal income tax progressivity, but their focus on closed-form solutions and analytic results force their model to be more stylized in some aspects, such as the zero net wealth, one-shot investment in skills, and the absence of female earners. Kaymak and Poschke (2015) are interested in the tax policy change since 1960s in the U.S., but they focus on the economic consequences of exogenous top income tax cuts, whereas I consider the determination of the tax policy in response to economic and non-economic changes. There is also a literature asking the optimal tax policy questions using the mechanism design approach such as Farhi and Werning (2013) and most recent Stantcheva (2015) with human capital. This approach allows more flexible tax system, but the optimization problem is typically difficult to solve in dynamic or non-standard contexts. Also, whether such flexible and hence more complicated tax system is feasible in practice remains a question.

The numerical method developed in this paper to infer the Pareto weights from the ac-

tual income tax policy is related to the so-called “inverse-optimum” research. Bourguignon and Spadaro (2012) first derive the formula to reverse the static optimal income tax problem, and use it to infer the Pareto weights from actual marginal income tax rates in France. Lockwood and Weinzierl (2014) use the same method and apply it to the U.S. data. Their method is more restrictive in the sense that it requires the utility function to be quasi-linear and can only be applied to stylized static model, whereas my numerical method can be applied to more general preferences and quantitative dynamic models. Chang, Chang, and Kim (2015) conduct a cross-country study to uncover the Pareto weights with similar spirit, but they only consider flat income tax and do not have human capital accumulation and female earners in their model.

The rest of this paper is organized as follows. Section 2.2 presents the quantitative life-cycle model, the Ramsey optimal tax policy problem, and the numerical method to invert the Ramsey problem. Section 2.3 describes the calibration strategy and reports the calibration results. Section 2.4 presents the quantitative analysis and results. Section 2.5 discusses possible explanations for the change in Pareto weights from a political economy point of view. Section 2.6 concludes.

## 2.2 Model

In this section, I present the quantitative life-cycle model employed to capture the changes in economic fundamentals between the 1970s and 2010s. I first describe the problems of households, the representative firm, and the government and define the stationary competitive equilibrium. Then I formalize the Ramsey optimal tax policy problem. Finally, I describe the method used to infer the Pareto weights from the actual tax policy.

## 2.2.1 Households

The economy is populated with overlapping generations households. In each year, a measure one of new households are born at age 1. Each household consists of two members: a male and a female. Both members can work in the first  $T$  years of the household's life cycle, then enter retirement for another  $T_R$  years, and then die for sure.<sup>9</sup> I use subscript  $i$  and  $j$  to denote the gender and age of an earner, with  $i = 1$  and  $2$  corresponding to the male and the female, and omit the index for different households for simplicity.

Within each cohort, households are heterogeneous at birth in their learning abilities  $\{A_i\}_{i=1}^2$ . To reduce the number of state variables, I assume that the male and female learning abilities  $\{A_i\}_{i=1}^2$  are determined by a common household level ability variable  $A$ , and  $A_i = f_i(A)$ ,  $f'_i(\cdot) > 0$ ,  $i = 1, 2$ . Hence the high ability males are matched with the high ability females. The ability of an earner is constant over the life cycle, and the distribution of household level learning ability  $A$ , governed by the cdf  $F(A)$ , is assumed to be the same across cohorts. The initial productivity of an earner  $w_{i,1}$  is positively correlated with his or her ability  $A_i$  but not perfectly.<sup>10</sup>

A Ben-Porath style human capital investment technology is available to all the earners, which allows an earner to increase his or her productivity  $w_{i,j+1}$  by spending some time  $n_{i,j}$  studying at age  $j$ . In addition to the study time, the outcome of such investment also depends on the learning ability of the earner  $A_i$ , the productivity at current age  $w_{i,j}$ , the rate of return to human capital investment  $R_H^i$ , and the realization of an idiosyncratic human capital shock  $z_{i,j+1}$ :

$$w_{i,j+1} = e^{z_{i,j+1}} [w_{i,j} + R_H^i A_i (w_{i,j} n_{i,j})^\alpha]. \quad (2.2.1)$$

<sup>9</sup>Unfortunately, I have to abstract away from the marriage and divorce processes because modeling those requires keeping track of equilibrium distributions of single males and females, which is computationally challenging given the number of state variables in the model.

<sup>10</sup>Huggett, Ventura, and Yaron (2011) estimate that the correlation between the log ability and the log initial productivity is about 0.8.

The main difference from the standard Ben-Porath formula is the additional parameter  $R_H^i$ , which is used to capture the rising return to human capital investment between the 1970s and 2010s as skill-biased technological change.<sup>11</sup> The idiosyncratic human capital shocks  $\{z_{i,j+1}\}_{i=1}^2$  are allowed to be correlated between the two earners within each household, but are i.i.d. over time and across households. The means of these shocks are slightly negative and represent the depreciation of human capital over time.

The working-age households can earn labor income from the labor supply of the male and the female. Let  $\tilde{w}_{i,j}$  and  $l_{i,j}$  denote the wage rate and the time worked of the gender  $i$  earner at age  $j$ , then the before-tax labor income of this earner is  $y_{i,j} = \tilde{w}_{i,j}l_{i,j}$ . The wage rate  $\tilde{w}_{i,j}$  is determined by three components:  $\tilde{w}_{i,j} = w_e\theta_i w_{i,j}$ , where  $w_{i,j}$  is the productivity of the earner,  $\theta_i$  is a gender factor of labor productivity, and  $w_e$  is the wage rate of effective labor. The gender factor for males  $\theta_1$  is normalized to be 1, while  $\theta_2$  for females is used to capture the increase of female labor productivity between the 1970s and 2010s. The wage rate of effective labor  $w_e$  can also be different between the two time periods to reflect the change of the overall labor productivity.<sup>12</sup>

Besides labor income, households can earn capital income by saving in a risk-free bond with interest rate  $r$ , but cannot borrow into negative asset positions. The retired households also receive social security benefits  $b$  from the government in each retirement year. There are no insurance markets for the idiosyncratic human capital shocks, so the financial markets are incomplete.

Only labor income is taxed by the government, and the tax policy is summarized by the

<sup>11</sup>The law of motion for  $w_{i,j}$  here follows the human capital investment technology in Guvenen and Kuruscu (2010) and Guvenen, Kuruscu, and Ozkan (2014). They assume further that the labor productivity  $w_{i,j}$  is determined by two components: raw labor and human capital. In that case,  $R_H^i$  is the price of human capital, and an increase of  $R_H^i$  corresponds to skill-biased technological change. More details are available in those two papers.

<sup>12</sup>The introduction of  $w_e$  and  $\theta_2$  allows the model to match the levels and the shapes of male and female labor income life cycles at the same time. The same effects can be obtained by allowing the distribution of learning ability to be different over time, which is a less appealing assumption to make.



function  $T(\cdot)$  which gives the tax liability based on the household's total before-tax labor income. Hence, the after-tax labor income of a household with before-tax labor income  $\{y_{i,j}\}_{i=1}^2$  is  $\sum_{i=1}^2 y_{i,j} - T(\sum_{i=1}^2 y_{i,j})$ .

The state variables of a working-age household are the savings  $a$ , the male and female labor productivities  $\{w_i\}_{i=1}^2$ , the age of the household  $j$ , and the household ability level  $A$ . In each year, the two earners in each household make joint decisions on current household consumption, savings, labor supply, and study time. They enjoy consumption, but dislike non-leisure time of work and study. Hence, a working-age household's problem is in the recursive form:

$$V(a, w_1, w_2, j, A) = \max_{\{c, a', (l_i, n_i, w'_i)_{i=1}^2\}} u(c, l_1, l_2, n_1, n_2) + \beta \sum_{z'} \pi(z') V(a', w'_1, w'_2, j + 1, A)$$

s.t.

$$c + a' = \left[ \sum_{i=1}^2 y_i - T\left(\sum_{i=1}^2 y_i\right) \right] + (1 + r)a;$$

$$w'_i = e^{z'_i} [w_i + R_H^i A_i (w_i n_i)^\alpha], i = 1, 2;$$

$$z' = (z'_1, z'_2)^T \sim i.i.d. N(\mu_z, \Sigma_z);$$

$$A_i = f_i(A), i = 1, 2;$$

$$y_i = w_e \theta_i w_i l_i, i = 1, 2;$$

$$a' \geq 0, l_i \geq 0, c \geq 0, n_i \geq 0.$$

where  $\beta$  is the discount factor of future utility, and  $\pi(\cdot)$  is the joint pdf of the idiosyncratic human capital shocks.

Members of a retired household no longer work or study and suffer no risks. Hence the state variables of a retired household are only the savings  $a$  and the age of the household  $j$ ,

and its problem in the recursive form is:

$$V^R(a, j) = \max_{\{c, a'\}} u^R(c) + \beta V^R(a, j + 1)$$

s.t.

$$c + a' = b + (1 + r)a;$$

$$a_{(T+T_R)} \geq 0, c \geq 0.$$

The instantaneous utility function of working-age households is assumed to be additive separable between consumption and non-leisure time and takes the following functional form:

$$u(c, l_1, l_2, n_1, n_2) = \log(c) - \psi_1 \frac{(l_1 + n_1)^{1+\frac{1}{\eta_1}}}{1+\frac{1}{\eta_1}} - \psi_2 \frac{(l_2 + n_2)^{1+\frac{1}{\eta_2}}}{1+\frac{1}{\eta_2}}$$

where  $\eta_i$  is the earner  $i$ 's Frisch elasticity of labor supply, and  $\psi_i$  captures the level of disutility from earner  $i$ 's non-leisure time. The instantaneous utility function of retired households is simply:

$$u^R(c) = \log(c).$$

## 2.2.2 Representative Firm

The production side of the economy consists of measure one profit-maximizing perfect competitive firms. They rent physical capital at interest rate  $r$  and hire effective labor at wage rate  $w_e$  to produce the final good used for both consumption and investment in physical capital. All the firms have the same production technology, which takes the standard Cobb-Douglas functional form:

$$Y = K^\omega (Z\tilde{L})^{1-\omega}$$

where  $Y$  is the final output,  $K$  is the physical capital rented,  $\tilde{L}$  is the effective labor hired,  $Z$  is the productivity of effective labor, and  $\omega$  is the parameter governing the capital income share. Because this production function has constant return to scale, all the firms make zero profits at the competitive equilibrium, and the entire production side is equivalent to one representative firm who takes the input and output prices as given and maximizes its profits period by period. The representative firm's problem is then

$$\max_{\{K, \tilde{L}\}} K^\omega (Z\tilde{L})^{1-\omega} - (1+r)K - w_e\tilde{L} + (1-\delta)K$$

where  $\delta$  is the depreciation rate of physical capital. The optimality conditions of the representative firm are then:

$$r = \omega \left( \frac{K}{Z\tilde{L}} \right)^{\omega-1} - \delta,$$

$$w_e = (1-\omega)Z \left( \frac{K}{Z\tilde{L}} \right)^\omega.$$

### 2.2.3 Government

The government only levies labor income tax at household level, and the tax liability only depends on the total before-tax labor income of the household, which is given by the function  $T(\cdot)$ . There are two uses of the tax revenues by the government: (1) paying the total social security benefits to the retired households,  $T_R b$ ; (2) financing government services,  $G$ . Hence the labor income tax in the model corresponds to the combination of labor income tax and social security tax in the U.S.

Government can only choose the level of  $G$  to balance its budget period by period for a

given tax function  $T(\cdot)$  and the level of retirement benefits  $b$ :

$$\sum_{j=1}^T \int T\left(\sum_{i=1}^2 y_{i,j}(s)\right) d\Phi_j(s) = T_R b + G$$

where  $s$  is the vector of household state variables except for age, i.e.,  $s = (a, \{w_i\}_{i=1}^2, A)$  when  $j = 1, \dots, T$ , and  $s = a$  when  $j = T + 1, \dots, T + T_R$ ;  $y_{i,j}(s)$  is the earner  $i$ 's labor income in an age  $j$  household with state  $s$ , and  $\Phi_j(s)$  gives the measure of households with age  $j$  and state  $s$ .

Following Bénabou (2002) and Heathcote, Storesletten, and Violante (2014), the labor income tax function  $T(\cdot)$  is assumed to take the form of

$$T(y) = y - (1 - \tau)y^{1-\mu} \quad (2.2.2)$$

where  $\tau$  and  $\mu$  are parameters governing the level and progressivity of the income tax. With this tax function,  $1 - \mu$  is the elasticity of after-tax income with respect to before-tax income. If  $\mu = 0$ , this elasticity is one, and the income tax rate is flat. The larger is the value of  $\mu$ , the smaller is this elasticity, which means that the income tax is more progressive. The parameter  $\tau$  on the other hand affects only the level of income tax and has no impacts on this elasticity.

## 2.2.4 Stationary Competitive Equilibrium

The economy is assumed to be open with free capital movement across the border, hence the domestic interest rate  $r$  is fixed at the global level  $r^*$ .<sup>13</sup> In the exercises of this

<sup>13</sup>While the U.S. economy is not the typical small open economy people often have in their minds, it is not a closed economy, either. Studies such as Warnock and Warnock (2009) have shown that the interest rates in the U.S. are significantly affected by the flows of foreign capital, and a large portion of the U.S. government debts are held by foreigners. The key implication of this assumption is that the domestic interest rate will not respond to the changes of income tax policy, which greatly reduces the computation burden.

paper, I focus on the stationary competitive equilibrium of the economy, which is defined in the following.

**Definition of Stationary Competitive Equilibrium:** A stationary competitive equilibrium is a collection of household value and policy functions  $\{V, V^R, c, a', (l_i, n_i)_{i=1}^2\}$ , the representative firm's decisions  $\{K, \tilde{L}\}$ , government expenditure  $G$ , the wage of effective labor  $w_e$ , the domestic interest rate  $r$ , and a sequence of distributions of household state  $\{\Phi_j(s)\}_{j=1}^{T+T_R}$  such that

1. Households: given the prices  $\{w_e, r\}$ , the tax function  $T(\cdot)$ , and the social security benefits  $b$ , the collection of household value and policy functions  $\{V, V^R, c, a', (l_i, n_i)_{i=1}^2\}$  solves the household's problem.
2. Representative firm: given the prices  $\{w_e, r\}$ , the values of  $\{K, \tilde{L}\}$  satisfy the representative firm's optimality conditions.
3. Government: given the tax function  $T(\cdot)$ , social security benefits  $b$ , and the household policy functions, the value of  $G$  satisfies the government budget constraint.
4. The labor market clears:

$$\tilde{L} = \sum_{j=1}^T \int \sum_{i=1}^2 \theta_i w_{i,j}(s) l_{i,j}(s) d\Phi_j(s).$$

5. The physical capital market clears:

$$r = r^*.$$

6. Stationary conditions: given  $\Phi_1(\cdot)$ , the law of motion of  $\{\Phi_j(\cdot)\}_{j=1}^{T+T_R}$  induced by the household policy functions, demographics, and idiosyncratic shocks,  $\{H_j(\cdot)\}_{j=1}^{T+T_R-1}$ ,

satisfies

$$\Phi_{j+1} = H_j(\Phi_j), j = 1, \dots, T + T_R - 1.$$

### 2.2.5 Ramsey Optimal Tax Policy Problem

A Ramsey government chooses the labor income tax function  $T(\cdot)$  to solve the following optimization problem at the stationary competitive equilibrium.

$$\begin{aligned} \max_{\{T(\cdot), G\}} & \int \left[ \int V(a_0, w_{1,1}, w_{2,1}, 1, A) d\Pi(a_0, w_{1,1}, w_{2,1} | A) \right] W(A) dF(A) \\ & + \gamma \left( \sum_{j=1}^{T+T_R} \beta^{j-1} \right) \log \left( \frac{G}{T + T_R} \right) \end{aligned}$$

s.t.

$$\sum_{j=1}^T \int T \left( \sum_{i=1}^2 y_{i,j}(s) \right) d\Phi_j(s) = T_R b + G$$

The first part of the Ramsey government's objective function is a weighted sum of expected lifetime utility of a newborn cohort at the stationary competitive equilibrium. In particular,  $V(a_0, w_{1,1}, w_{2,1}, 1, A)$  is the expected lifetime utility of a newborn ability  $A$  household with the initial state  $(a_0, w_{1,1}, w_{2,1})$ ;  $\Pi(a_0, w_{1,1}, w_{2,1} | A)$  is the conditional cdf of initial household state  $(a_0, w_{1,1}, w_{2,1})$  given the household ability  $A$ ;  $W(A)$  is the Pareto weight assigned to the ability  $A$  households;<sup>14</sup> and  $F(A)$  is the unconditional cdf of the household ability  $A$ . If  $W(A)$  equals to one for all values of  $A$ , it becomes utilitarian weight function, and the first part is simply the expected lifetime utility of a newborn household at the stationary competitive equilibrium before any uncertainty is resolved.

<sup>14</sup>In general, the Pareto weight function  $W(\cdot)$  can be a function of all the household state variables. However, for tractability, I restrict it to be a function of only the household learning ability  $A$ . This assumption reduces the identification burden significantly when inferring the Pareto weight function from the actual tax policy. Since  $A$  is directly related to the expected lifetime income of households at equilibrium, this weight function allows us to capture the government's preference over households with different lifetime income.

The second part of the Ramsey government's objective function is the lifetime utility of a newborn cohort generated by government services at the stationary competitive equilibrium.  $G$  is the total government services provided in each year, and  $\frac{G}{T+T_R}$  is the government services per household.<sup>15</sup> The flow utility of each household generated by government services is assumed to be the log of government services per household,<sup>16</sup> and it is discounted by the same discount factor  $\beta$  as the household private flow utility. The parameter  $\gamma$  is the Pareto weight on government services. Obviously, the scale of the weight function  $W(A)$  and  $\gamma$  does not matter, so we can normalize  $\gamma = 1$  for identification.

## 2.2.6 Inverting the Ramsey Problem

For the purpose of this paper, we need to find the Pareto weight function which can rationalize the actual income tax policy we observe, i.e, the Ramsey government would choose the actual income tax policy with such Pareto weight function. One way of doing this is to use the first order conditions of the Ramsey problem.

Suppose the observed income tax function  $T(\cdot)$  is parameterized with  $M$  parameters  $\{\tau_m\}_{m=1}^M$ , then the income tax policy is represented by the values of  $\{\tau_m\}_{m=1}^M$ .<sup>17</sup> We can then parameterize the weight function as  $W(A) = \sum_{p=1}^M \xi_p F(A)^{p-1}$ , where  $F(A)$  is the cdf of household ability  $A$ ,<sup>18</sup> and  $\{\xi_p\}_{p=1}^M$  are coefficients to be determined. The first order

<sup>15</sup>Recall that there are measure one households in each cohort, so the total measure of households at any given year is  $T + T_R$ .

<sup>16</sup>The choice of the log function is such that when the model economy is scaled up, the government expenditure share would remain stable relative to the size of the economy.

<sup>17</sup>In the exercises of this paper, the income tax function only has two parameters,  $\tau$  and  $\mu$ , as in Equation (2.2.2). I describe the method here in the most general form to show that it can be applied to more flexible income tax functions, and hence can allow more flexible Pareto weight functions.

<sup>18</sup>Other basis functions can also be used to parameterize the weight function. The advantage of using the cdf  $F(A)$  here is such that  $W(A)$  is bounded even if the distribution of  $A$  is unbounded.

conditions of the Ramsey problem are then

$$\int \left[ \int \frac{\partial V(a_0, w_{1,1}, w_{2,1}, 1, A)}{\partial \tau_m} d\Pi(a_0, w_{1,1}, w_{2,1} | A) \right] \left[ \sum_{p=1}^M \xi_p F(A)^{p-1} \right] dF(A)$$

$$= - \left( \sum_{j=1}^{T+T_R} \beta^{j-1} \right) \frac{1}{G} \frac{\partial G}{\partial \tau_m}, m = 1, \dots, M.$$

where the amount of government services  $G$  is treated as an implicit function of the income tax policy  $\{\tau_m\}_{m=1}^M$ , which is defined by the government budget constraint. To simplify the notations, let  $B_{m,p} = \int \int \frac{\partial V(a_0, w_{1,1}, w_{2,1}, 1, A)}{\partial \tau_m} d\Pi(a_0, w_{1,1}, w_{2,1} | A) F(A)^{p-1} dF(A)$ , then the above system of equations becomes:

$$\begin{bmatrix} B_{1,1} & \cdots & B_{1,M} \\ \vdots & \ddots & \vdots \\ B_{M,1} & \cdots & B_{M,M} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \vdots \\ \xi_M \end{bmatrix} = - \left( \sum_{j=1}^{T+T_R} \beta^{j-1} \right) \frac{1}{G} \begin{bmatrix} \frac{\partial G}{\partial \tau_1} \\ \vdots \\ \frac{\partial G}{\partial \tau_M} \end{bmatrix}$$

All the  $B_{m,p}$  and  $\frac{\partial G}{\partial \tau_m}$  can be computed numerically using the economic model at the observed values of  $\{\tau_m\}_{m=1}^M$ , so we can simply solve this linear system of equations for the coefficients  $\{\xi_p\}_{p=1}^M$  and hence recover the Pareto weight function.

Note that the number of coefficients in the Pareto weight function is intentionally chosen to be exactly the same as the number of parameters in the tax function to guarantee the existence and uniqueness of the inferred Pareto weight function. We also need the objective function to be concave with respect to the choice variables  $\{\tau_m\}_{m=1}^M$  for the sufficiency of the first order conditions and the  $(B_{m,p})$  matrix to be invertible, both of which can be verified numerically.



## 2.3 Calibration

In this section, I describe the calibration strategy for the economic model and report the calibrated parameter values and the empirical targets matched by the model. To choose the values of parameters, an economy at the stationary competitive equilibrium with 20000 households in each cohort is simulated from the model and first calibrated to match the U.S. economy in the years 2010-2012 for the early 2010s. For the late 1970s, part of the parameters are kept the same as in the years 2010-2012 such as those governing the household preference and the distribution of learning ability, whereas the others are recalibrated to match the 1978-1980 empirical targets such as the return to human capital investment and the female factor of labor productivity, etc. The data used to compute the empirical targets are from the Annual Social and Economic Supplement (ASEC) of the Current Population Survey (CPS) and the core sample of the Panel Study of Income Dynamics (PSID). All the nominal variables are deflated using the Bureau of Labor Statistics' Consumer Price Index Research Series (CPI-U-RS).

### 2.3.1 Calibration Strategy

In the model, one unit of the final good represents \$58026 in 2012 dollars, which is the mean labor income of a working male with age 23 to 65 in the years 2010-2012. The empirical life cycle profiles used are cross-sectional life cycles, i.e, they are computed from the CPS cross-sectional data for households at different ages. Since the upper half of the income distribution is more important for the income tax policy questions, the calibration of the model focuses on matching the life cycle profiles of the 50th, 90th and 99th percentiles of the male and female labor income in addition to a selected group of empirical moments. Many parameters are jointly selected to match these empirical targets at the same time, hence there are no exact one-to-one mappings between the parameters and the empirical

targets. However, we can still link different parameters to the empirical targets which offer most of the identification powers for them.

**Demographics.** The starting age of households is set to be 23 and the retirement age is 65. Hence  $T$  is 43 in the model. For the years 2010-2012, the length of retirement,  $T_R$ , is set to be 14 years based on the life expectancy of 79 in the U.S.

**Preference and Interest Rate.** The Frisch elasticities of the male and female labor supply  $\eta_1$  and  $\eta_2$  are set at 0.4 and 0.8, respectively. These values are consistent with recent studies on family labor supply such as Blundell, Pistaferri, and Saporta-Eksten (2012). The parameter capturing the disutility of the male non-leisure time  $\psi_1$  is normalized to be 1 because the unit of time can be freely adjusted in the model. The female counterpart  $\psi_2$  is mainly identified by the female-male labor income ratio and hours worked ratio. The global risk-free real interest rate  $r^*$  is set at 1%. The discount factor of flow utility  $\beta$  is set at 0.99.

**Production Technology.** Because the economy is open with free capital movement across the border, the domestic interest rate  $r$  is fixed at the global level  $r^*$  across equilibria. Also, the ratio  $\frac{K}{ZL}$  is pinned down by  $r^*$ ,  $\delta$ , and  $\omega$  from the representative firm's optimality conditions, and it is not affected by changes of income tax policy. Therefore,  $w_e$  is also a constant across equilibria. So instead of calibrating the values of  $Z$ ,  $\omega$ , and  $\delta$ , we only need the values of  $r$  and  $w_e$  to solve the model. The value of  $w_e$  is identified by the mean labor income of working males.

**Gender Factors of Labor Productivity.** For males, the gender factor of productivity  $\theta_1$  is normalized to be 1. For females,  $\theta_2$  is identified from the female-male labor income and hours worked ratios together with  $\psi_2$ .

**Human Capital Accumulation Technology.** The parameter  $\alpha$  in the Ben-Porath human capital accumulation formula governs the curvature of the return to human capital investment. Its value is set at 0.7 such that the life cycle of the median male labor income

reaches its peak around age 55. The level of the return to human capital investment  $R_H^i$  is not separately identified from the scale of the learning ability, and hence it can be normalized to be any positive number for the years 2010-2012. I choose  $R_H^1 = R_H^2 = 0.05$  such that the scales of other variables are convenient.

**Distribution of Learning Ability.** The distribution of the household learning ability  $A$  is assumed to be a shifted Pareto-log-normal distribution, i.e.,  $A \sim PLN(\mu_A, \sigma_A^2, \lambda_A) + e^{\mu_A} Const_A$ .<sup>19</sup> Within each household, the ability of the male is determined by  $A_1 = f_1(A) = A$ . Since the scales of  $A_1$ ,  $w_{1,1}$ ,  $w_e$ , and  $\theta_2$  are not separately identified,<sup>20</sup> the median of  $A_1$  is normalized to be 1 by choosing a proper value of  $\mu_A$ . The standard deviation of the log-normal part  $\sigma_A$  and the parameter  $Const_A$  are chosen to match the life cycle profiles of the 90th and 99th percentiles of the male labor income in the years 2010-2012. The Pareto parameter  $\lambda_A$  is used to target the Pareto ratio of male labor income within the top 1%.<sup>21</sup> Since the female labor income is less dispersed than the male labor income in the upper half of the distribution, I assume

$$\log(A_2) = \{ \mathbb{1}[A \geq median(A)] \beta_1^{A_2} + \mathbb{1}[A < median(A)] \} \log\left(\frac{A}{median(A)}\right) + \beta_0^{A_2}.$$

The scales of  $A_2$ ,  $w_{2,1}$ , and  $\theta_2$  are not separately identified, hence the median of  $A_2$  is also normalized to be 1 by setting  $\beta_0^{A_2} = 0$ . The value of  $\beta_1^{A_2}$  is chosen to match the life cycles of the 50th, 90th and 99th percentiles of the female labor income in the years 2010-2012.

**Initial Asset and Labor Productivity.** The initial asset at birth  $a_0$  is assumed to be zero for all households. The initial labor productivity of a gender  $i$  earner,  $w_{i,1}$ , is positively

<sup>19</sup>If  $x_1 \sim LN(\mu, \sigma^2)$  and  $x_2 \sim Pareto(\lambda)$ ,  $x_3 = x_1 x_2 \sim PLN(\mu, \sigma^2, \lambda)$ .

<sup>20</sup>If we multiply  $A_1$  by  $x^{1-\alpha}$ ,  $w_{1,1}$  by  $x$ ,  $w_e$  by  $1/x$ ,  $\theta_2$  by  $x$ , and keep all the other parameters the same, the economy will be exactly the same as before.

<sup>21</sup>The Pareto ratio at a cutoff income level  $y_{cutoff}$  is defined as  $\frac{E(y|y \geq y_{cutoff})}{y_{cutoff}}$ . This is a measure of the shape of the income distribution at the upper tail.

correlated with his or her learning ability, and it is determined by

$$\log(w_{i,1}) = \beta_1^{w_i} \log(A_i) + \beta_0^{w_i} + \varepsilon_i$$

where  $\varepsilon_i \sim N(0, \sigma_{\varepsilon_i}^2)$  is the idiosyncratic shock to the gender  $i$  earner's initial productivity. The values of  $\beta_1^{w_i}$ ,  $\beta_0^{w_i}$  and  $\sigma_{\varepsilon_i}$  are calibrated to best match the life cycle profiles of the 50th, 90th and 99th percentiles of the male and the female labor income and the correlation between log initial productivity and log learning ability estimated by Huggett, Ventura, and Yaron (2011), which is about 0.8.

**Human Capital Shocks, Transitory Shocks and Measurement Errors.** The idiosyncratic shocks associated with the human capital accumulation  $z_j = (z_{1,j}, z_{2,j})^T$  are assumed to be i.i.d. over time and across households, but they are joint-normal distributed within each household between the male and the female.

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \sim N \left( \begin{bmatrix} \mu_{z_1} \\ \mu_{z_2} \end{bmatrix}, \begin{bmatrix} \sigma_{z_1}^2 & \rho_{z_1 z_2} \sigma_{z_1} \sigma_{z_2} \\ \rho_{z_1 z_2} \sigma_{z_1} \sigma_{z_2} & \sigma_{z_2}^2 \end{bmatrix} \right)$$

The mean of the joint-normal distribution of human capital shocks  $(\mu_{z_1}, \mu_{z_2})^T$  is chosen to match the decline of the life cycle income profiles near the retirement. The human capital shocks are permanent income shocks given their ways of entering the human capital accumulation formula. To match the earnings dynamics in the data, I multiply the labor income by another i.i.d. transitory income shocks,  $\varepsilon_{ts,i} \sim LN(0, \sigma_{ts,i}^2)$ , when calibrating the model. These transitory income shocks are assumed to be fully insurable through risk sharing among households, and therefore they have no effects on household behaviors.<sup>22</sup> Also, the micro level income survey data usually bear nontrivial measurement er-

<sup>22</sup>The main benefit of this assumption is to avoid adding two additional state variables to the household's problem. It should not affect the results much as previous empirical estimates and model simulation results all suggest that households can attain almost perfect insurance (> 90%) against transitory shocks (Blundell, Pistaferri, and Preston (2008), Kaplan and Violante (2010)).

rors. Hence, whenever compared with the empirical targets from CPS or PSID, the income data simulated from the model are multiplied by an i.i.d. measurement error component  $\varepsilon_{me} \sim LN(0, \sigma_{me}^2)$ . The standard deviation  $\sigma_{me}$  is set at 0.15.<sup>23</sup>

The covariance matrix of the human capital shocks and the variances of the transitory income shocks are calibrated jointly to match the earnings dynamics in the PSID data. The earnings dynamics are captured by the variances and first-order autocovariances of the male and female residual labor income growth, and the correlation between the male and female residual labor income growth within households. To compute these empirical moments from the PSID data, I first regress the log labor income on a group of age and year dummies to estimate the “life cycle” components of labor income and the time effects for each gender. The residual log labor income  $\hat{y}_{i,j}$  is computed by subtracting the “life cycle” components and the time effects from the actual log labor income,  $\log(y_{i,j})$ . Because the PSID data are biennial since the year 1997, the income growth is calculated over a two-year span, i.e.,  $\Delta\hat{y}_{i,j} = \hat{y}_{i,j} - \hat{y}_{i,j-2}$ , and the empirical targets are  $var(\Delta\hat{y}_{i,j})$ ,  $cov(\Delta\hat{y}_{i,j}, \Delta\hat{y}_{i,j-2})$ , and  $corr(\Delta\hat{y}_{1,j}, \Delta\hat{y}_{2,j})$ .<sup>24</sup>

**Taxable Income.** Because a half of the Federal Insurance Contributions Act (FICA) tax for Social Security and Medicare is paid by the employers and is not counted as a part of the taxable income of employees, the before-tax income in the data is different from the total income of employees. To account for this difference, all the income data simulated from the model are transformed into comparable before-tax income based on the FICA tax rate schedule before any statistics are calculated.

<sup>23</sup>The typical value of  $\sigma_{me}$  assumed in the literature ranges from 0.15 to 0.20.

<sup>24</sup>For the years 2010-2012 and 1978-1980, I use the PSID data from 1998-2012 and 1971-1980, respectively. The sample is restricted to the married male and female with the head age between 30 and 55. The observations with a residual income growth larger than 400% or smaller than -80% are excluded. The same process for the PSID data is also applied to the model-simulated data.

**Income Tax Function.** The labor income tax function takes the form of

$$T(y) = y - (1 - \tau)y^{1-\mu}$$

where  $\tau$  and  $\mu$  are measures of the level and progressivity of the income tax schedule. To pin down the values of  $\tau$  and  $\mu$  for the U.S., I use the NBER's TAXSIM program to create a mapping between a household's total income,  $y$ , and its total liability of the federal income tax and FICA tax,  $T(y)$ , based on the actual U.S. tax policy. The employer's share of the FICA tax is included in the total income and the total tax liability. Then  $\tau$  and  $\mu$  can be derived from the coefficients of the following equation, which can be estimated using the OLS method:

$$\log(y - T(y)) = \log(1 - \tau) + (1 - \mu) \log(y).$$

**Social Security Benefits.** The social security benefits  $b$  in the model are chosen to be the sum of the average male and female social security benefits in the U.S.

**The Years 1978-1980.** When calibrating the model to match the U.S. economy in the years 1978-1980, four sets of parameters are recalibrated to reflect the changes in economic fundamentals: (1) the length of retirement  $T_R$  to reflect the change of life expectancy; (2) the return to human capital investment  $R_H^i$ <sup>25</sup>, the female factor of labor productivity  $\theta_2$ , and the wage of effective labor  $w_e$  to reflect the changes of the production and human capital accumulation technologies; (3) the parameters governing the dispersion of initial productivity,  $\beta_1^{w_i}$  and  $\sigma_{\varepsilon_i}^2$ , the covariance matrix of human capital shocks  $\Sigma_z$ , and the variances of transitory income shocks  $\sigma_{ts,i}^2$  to capture the changes of idiosyncratic risk; (4) the income tax function parameters  $\tau$  and  $\mu$ , the FICA tax rates, and the social security benefits  $b$  to account for the changes of the income tax and social security policies. Other parameters

<sup>25</sup>  $R_H^i$  can now be separated from  $A_i$  because the learning ability is assumed to be the same between the years 1978-1980 and 2010-2012.

are kept the same as those for the years 2010-2012. Because less parameters need to be calibrated, some empirical targets are not used in the calibration for the years 1978-1980, for example, the female-male hours worked ratio and the Pareto ratio at the top 1% of the male labor income distribution.

## 2.3.2 Calibration Results

### Parameter Values

Table 2.1 provides the calibrated parameter values for the years 2010-2012.<sup>26</sup> The female learning ability is less dispersed than the male in the upper half of the distribution as the value of  $\beta_1^{A_2}$  is less than one. Compared with the male earners, the female earners face smaller initial labor productivity risk but larger human capital risk over the life cycle. The human capital shocks between the two earners of each household are positively correlated with a correlation coefficient of 0.335. The depreciation rate of human capital is about 2% per year. The magnitudes of transitory income risk are similar for both genders. The income tax policy in the years 2010-2012 implies that the elasticity of after-tax income with respect to before-tax income is about 0.914 as  $\mu$  equals 0.086. The social security benefits for the years 2010-2012 are 0.416 per retired household per year, which is about \$24139 in 2012 dollars.

The changes of parameter values between the years 1978-1980 and 2010-2012 are reported in Table 2.2. From the 1970s to 2010s, the length of retirement  $T_R$  increases from 9 to 14 as a result of the extended life expectancy in the U.S. from 74 to 79. The rise of return to human capital investment  $R_H^i$  and the increase of female factor of labor productivity  $\theta_2$  reflect skill-biased technological change and increased female labor productivity. Com-

<sup>26</sup>Although the female factor of labor productivity  $\theta_2$  is greater than one here, it does not mean females are more productive than males because the total productivity is determined by  $\theta_i w_{i,j}$ . The value of  $\theta_2$  could be different depending on the normalization choice on the level of learning ability.

Table 2.1: Calibrated Parameter Values (2010-2012)

Category	Related Formula	Parameters	Values
Demographic	$j = 1, \dots, T, \dots, (T + T_R)$	$(T, T_R)$	(43, 14)
Preference	$\log(c) - \psi_1 \frac{(l_1+n_1)^{1+\frac{1}{\eta_1}}}{1+\frac{1}{\eta_1}} - \psi_2 \frac{(l_2+n_2)^{1+\frac{1}{\eta_2}}}{1+\frac{1}{\eta_2}}$	$(\eta_1, \eta_2, \psi_1, \psi_2)$	(0.4, 0.8, 1, 0.82)
Interest Rate	$r = r^*$	$(r^*, r)$	(0.01, 0.01)
Discount Factor	$V(s, j) = \max u + \beta E[V(s', j + 1)]$	$\beta$	0.99
Wage and Gender Factor	$y_{i,j} = w_e \theta_i w_{i,j} l_{i,j}$	$(w_e, \theta_1, \theta_2)$	(1.93, 1, 1.07)
Human Capital Technology	$w'_i = e^{z_i} [w_i + R_H^i A_i (w_i n_i)^\alpha]$	$(\alpha, R_H^1, R_H^2)$	(0.7, 0.05, 0.05)
Distribution of Learning Ability	$A \sim PLN(\mu_A, \sigma_A^2, \lambda_A) + e^{\mu_A} Const_A$ $A_1 = A, A_2 = f_2(A; \beta_1^{A_2}, \beta_0^{A_2})$	$(\mu_A, \sigma_A, \lambda_A, Const_A)$ $(\beta_1^{A_2}, \beta_0^{A_2})$	(-1.86, 0.4, 2.5, 5) (0.70, 0)
Initial Asset	$a_0$	$a_0$	0
Initial Labor Productivity	$\log(w_{i,1}) = \beta_1^{w_i} \log(A_i) + \beta_0^{w_i} + \varepsilon_i$ $\varepsilon_i \sim N(0, \sigma_{\varepsilon_i}^2)$	$(\beta_1^{w_1}, \beta_0^{w_1}, \sigma_{\varepsilon_1})$ $(\beta_1^{w_2}, \beta_0^{w_2}, \sigma_{\varepsilon_2})$	(2.0, -0.85, 0.24) (1.8, -1.15, 0.16)
Human Capital Shocks	$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \sim N \left( \begin{bmatrix} \mu_{z_1} \\ \mu_{z_2} \end{bmatrix}, \begin{bmatrix} \sigma_{z_1}^2 & \rho_{z_1 z_2} \sigma_{z_1} \sigma_{z_2} \\ \rho_{z_1 z_2} \sigma_{z_1} \sigma_{z_2} & \sigma_{z_2}^2 \end{bmatrix} \right)$	$(\mu_{z_1}, \mu_{z_2})$ $(\sigma_{z_1}, \sigma_{z_2}, \rho_{z_1 z_2})$	(-0.02, -0.02) (0.109, 0.133, 0.335)
Transitory Shocks	$\varepsilon_{ts,i} \sim LN(0, \sigma_{ts,i}^2)$	$(\sigma_{ts,1}, \sigma_{ts,2})$	(0.110, 0.115)
Measurement Errors	$\varepsilon_{me} \sim LN(0, \sigma_{me}^2)$	$\sigma_{me}$	0.15
Income Tax Function	$T(y) = y - (1 - \tau)y^{1-\mu}$	$(\tau, \mu)$	(0.220, 0.086)
Social Security Benefits	$b$	$b$	0.416



Table 2.2: Parameter Changes between 1978-1980 and 2010-2012

Category	Related Formula	Parameters	1978-1980	2010-2012
Demographic	$j = 1, \dots, T, \dots, (T + T_R)$	$T_R$	9	14
Wage and Gender Factor	$y_{i,j} = w_e \theta_i w_{i,j} l_{i,j}$	$(w_e, \theta_2)$	(2.27, 0.77)	(1.93, 1.07)
Human Capital Technology	$w'_i = e^{z'_i} [w_i + R_H^i A_i (w_i n_i)^\alpha]$	$(R_H^1, R_H^2)$	(0.035, 0.020)	(0.05, 0.05)
Initial Labor Productivity	$\log(w_{i,1}) = \beta_1^{w_i} \log(A_i) + \beta_0^{w_i} + \varepsilon_i$ $\varepsilon_i \sim N(0, \sigma_{\varepsilon_i}^2)$	$(\beta_1^{w_i}, \sigma_{\varepsilon_1})$ $(\beta_1^{w_2}, \sigma_{\varepsilon_2})$	(1.25, 0.15) (1.25, 0.11)	(2.0, 0.24) (1.8, 0.16)
Human Capital Shocks	$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \sim N \left( \begin{bmatrix} \mu_{z_1} \\ \mu_{z_2} \end{bmatrix}, \begin{bmatrix} \sigma_{z_1}^2 & \rho_{z_1 z_2} \sigma_{z_1} \sigma_{z_2} \\ \rho_{z_1 z_2} \sigma_{z_1} \sigma_{z_2} & \sigma_{z_2}^2 \end{bmatrix} \right)$	$(\mu_{z_1}, \mu_{z_2})$ $(\sigma_{z_1}, \sigma_{z_2}, \rho_{z_1 z_2})$	(-0.02, -0.02) (0.104, 0.153, 0.225)	(-0.02, -0.02) (0.109, 0.133, 0.335)
Transitory Shocks	$\varepsilon_{ts,i} \sim LN(0, \sigma_{ts,i}^2)$	$(\sigma_{ts,1}, \sigma_{ts,2})$	(0.130, 0.125)	(0.110, 0.115)
Income Tax Function	$T(y) = y - (1 - \tau)y^{1-\mu}$	$(\tau, \mu)$	(0.258, 0.144)	(0.220, 0.086)
Social Security Benefits	$b$	$b$	0.274	0.416

bined with the declining wage of effective labor  $w_e$  between the 1970s and 2010s, these changes are more favorable to the high ability, the experienced, and the female earners relative to the low ability, the young, and the male earners. In the 1970s, the dispersion of initial productivity is much smaller. For the human capital shocks, the variance increases for males but decreases for females between the 1970s and 2010s. The correlation between these shocks within households has become more positive, which increases the risk at household level. If the variance of measurement errors is the same in the 1970s and 2010s, the data imply that the variances of transitory shocks have decreased. The income tax policy in the 1970s is more progressive because the elasticity of after-tax income with respect to before-tax income is 0.856 compared with the value of 0.914 in the 2010s. The income tax level is also about 3.8% higher in the 1970s as measured by the value of  $\tau$ .

### Empirical Targets Matched

Table 2.3 reports the empirical moments targeted in calibration, which can be matched almost perfectly by the model. The empirical moments about (residual) income growth are directly linked to the amount of idiosyncratic risk over the life cycle, which is important for the study of optimal income tax policy. Previous literature in labor economics has de-

Table 2.3: Empirical Moments Matched

Empirical Target	2010-2012		1978-1980	
	Data	Model	Data	Model
Mean Male Income	1	1.007	0.865	0.867
Female-Male Income Ratio	0.647	0.639	0.377	0.379
Female-Male Hours Worked Ratio	0.830	0.833	–	–
Pareto Ratio of Male Income at Top 1%	1.839	1.882	–	–
Variance of Male Income Growth	0.129	0.133	0.123	0.126
Variance of Female Income Growth	0.153	0.152	0.205	0.203
Autocovariance of Male Income Growth	–0.036	–0.037	–0.040	–0.041
Autocovariance of Female Income Growth	–0.034	–0.035	–0.037	–0.038
Correlation of Male and Female Income Growth	0.033	0.035	–0.026	–0.024

Note: Only labor income is included. “Female-Male Hours Worked Ratio” and “Pareto Ratio of Male Income at Top 1%” are not used in the calibration for the years 1978-1980. The moments about income growth are calculated using the residual income as defined in Section 2.3.1.

veloped methods to estimate the amount of idiosyncratic risk in the data using only statistic models of income process with both a permanent component and a transitory component such as Moffitt and Gottschalk (1995), Blundell, Pistaferri, and Preston (2008). Following that approach, I also estimate the amounts of idiosyncratic risk using both the PSID data and the model-simulated data and show that they are consistent with each other. Suppose the income process is determined by

$$\log(y_{i,j,t}) = \mathbb{F}(i, j, t) + P_{i,j} + u_{i,j}$$

$$P_{i,j+1} = P_{i,j} + v_{i,j+1}$$

where  $\mathbb{F}(i, j, t)$  is the income trend determined by gender  $i$ , age  $j$ , and year  $t$ ;  $P_{i,j}$  is the permanent component of residual labor income;  $v_{i,j}$  is the permanent shock; and  $u_{i,j}$  is the transitory component. Suppose the permanent shocks are i.i.d. over time and across

Table 2.4: Permanent and Transitory Shocks (Data vs. Model)

	1970s		2000s	
	Data	Model	Data	Model
<i>A. Permanent Shocks</i>				
$\sigma_{v_1}^2$	0.0211	0.0225	0.0287	0.0299
$\sigma_{v_2}^2$	0.0662	0.0635	0.0419	0.0404
$\rho_{v_1 v_2}$	-0.0551	-0.0498	0.0663	0.0718
<i>B. Transitory Shocks</i>				
$\sigma_{u_1}^2$	0.0404	0.0406	0.0357	0.0366
$\sigma_{u_2}^2$	0.0365	0.0378	0.0345	0.0354

Note: The results reported have been converted to the values corresponding to the one-year interval.

households but are joint-normal distributed within households,

$$\begin{bmatrix} v_{1,j} \\ v_{2,j} \end{bmatrix} \sim N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{v_1}^2 & \rho_{v_1 v_2} \sigma_{v_1} \sigma_{v_2} \\ \rho_{v_1 v_2} \sigma_{v_1} \sigma_{v_2} & \sigma_{v_2}^2 \end{bmatrix} \right).$$

And the transitory components are just i.i.d white noises:  $u_{i,j} \sim N(0, \sigma_{u_i}^2)$ ,  $i = 1, 2$ . Let  $\tilde{y}_{i,j} = \log(y_{i,j,t}) - \mathbb{F}(i, j, t)$  and  $\Delta \tilde{y}_{i,j} = \tilde{y}_{i,j} - \tilde{y}_{i,j-1}$ , then it is easy to derive that

$$\begin{aligned} \text{var}(\Delta \tilde{y}_{i,j}) &= \sigma_{v_i}^2 + 2\sigma_{u_i}^2; \\ \text{cov}(\Delta \tilde{y}_{i,j+1}, \Delta \tilde{y}_{i,j}) &= -\sigma_{u_i}^2; \\ \text{cov}(\Delta \tilde{y}_{1,j}, \Delta \tilde{y}_{2,j}) &= \rho_{v_1 v_2} \sigma_{v_1} \sigma_{v_2}. \end{aligned}$$

Therefore, we can estimate the covariance matrix of the permanent shocks and transitory components from these moments. The results are reported in Table 2.4.<sup>27</sup> It is not a surprise that the model fits the data well in this aspect because the structural shocks in the model

<sup>27</sup>Note the PSID data are biennial after 1997, so the formula should be adjusted accordingly. The sample is restricted to the married male and female with the head age between 30 and 55 in the years 1971-1980 and 1998-2012.

are calibrated to match the empirical moments used in this estimation.

Figure 2.1 plots the life cycle profiles of the 50th, 90th, and 99th percentiles of the male and female labor income for the years 2010-2012 and 1978-1980 in the data and in the model. The model matches well the life cycle profiles of the male labor income in both the

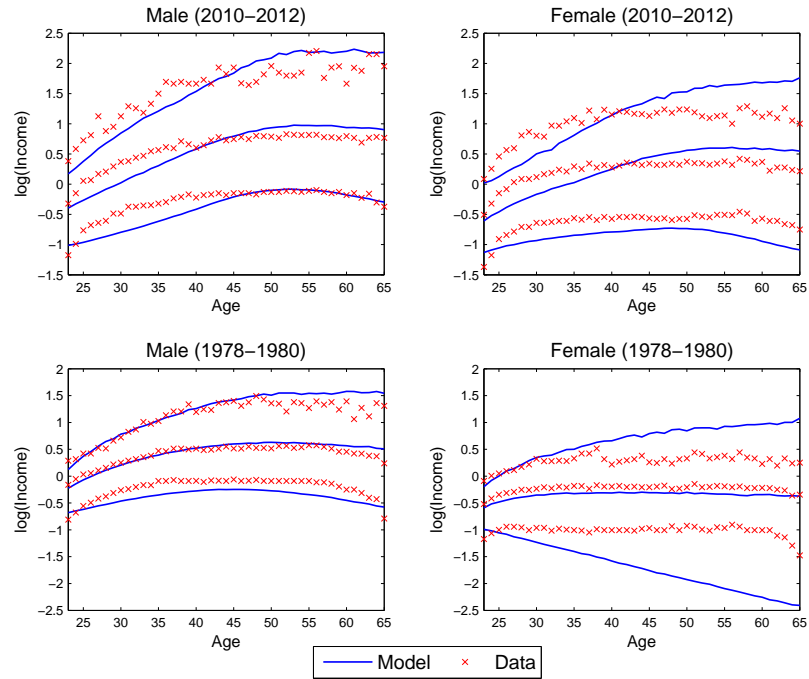


Figure 2.1: Matched Life Cycles of 50th, 90th, and 99th Percentiles of Labor Income

2010s and 1970s, which is a success of the model. For the female life cycle profiles, the differences between the model and the data are larger. This is partly because the assumption that the male and female learning abilities in each household are linked by the household level ability variable  $A$ , which limits the degrees of freedom in the distributions of the male and female learning abilities and therefore reduces the model's ability to match all the patterns in the data. However for the 1970s, the more significant differences of the female life cycle profiles between the data and the model are mainly driven by the differences between the CPS and PSID data. The PSID data imply a large variance of permanent income shocks for females in the 1970s, and hence a large variance of human capital shocks

for females in the model. As a result, even if there was no human capital accumulation at all, the cross-sectional dispersion of female labor income should rise rapidly with age as shown by the life cycle profiles in the model. But this is not the case in the life cycle profiles from the CPS data in the 1970s because CPS data are cross-sectional. My own interpretation of this fact is that the income processes of young females and old females were already very different in the 1970s. For those females who were near retirement in the late 1970s, their income when they entered the labor market was probably much lower (if not zero) than the income of young females in the late 1970s.<sup>28</sup>

Figure 2.2 plots the actual average income tax rates at different income levels together with the tax rates calculated from the fitted income tax functions for the years 2010-2012 and 1978-1980. As we can see, the income tax function can fit the actual tax schedule very well with the calibrated parameter values.

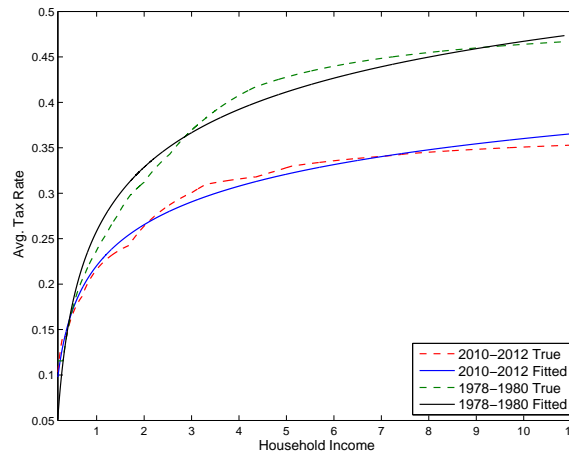


Figure 2.2: Matched Income Tax Functions

<sup>28</sup>One may then question whether we should use the cross-sectional life cycles to calibrate the model, or the life cycles from the same cohorts. Because the purpose of this paper is to study the income tax policy which mainly redistributes income cross-sectionally, I think the cross-sectional life cycles are the more appropriate calibration targets to use.

## 2.4 Quantitative Analysis and Results

In this section, I present the quantitative analysis and results based on the calibrated economic model. I first report the household life cycles in the model and show how household behaviors differ according to their learning abilities and between the 1970s and 2010s. Then I report the inferred Pareto weights from the actual income tax policies in the 1970s and 2010s. With the inferred Pareto weights in the 1970s, I compute the optimal response of the income tax policy with respect to *only* the changes in economic fundamentals since the 1970s and ascribe the remaining part of income tax policy change to the change in Pareto weights.<sup>29</sup> I also conduct a detailed decomposition of the income tax policy change with respect to each economic change in the model. Then I examine the sources of rising income inequality and labor income growth through the lens of my model and quantify the roles of income tax policy in those changes. Finally, I report the optimal income tax policy with utilitarian Pareto weights. All the results are based on simulated economies at the stationary competitive equilibrium with 20000 households in each cohort.

### 2.4.1 Household Life Cycles in the Model

Figure 2.3 plots the life cycles of three household groups in the model for the years 2010-2012. The three household groups are determined by their percentiles in the distribution of learning ability. In particular, those within the 10-percent intervals centered around the 10th, 50th, and 90th percentiles are selected, and the life cycle profiles plotted are the cross-sectional means within each group across ages.

Over the household life cycle, household consumption rises significantly at first due to increasing household income and binding borrowing constraints, but becomes relatively flat in the rest of the life cycle as a result of consumption smoothing. In the model, there are

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<sup>29</sup>Appendix 2.B provides sensitivity analysis for these results.

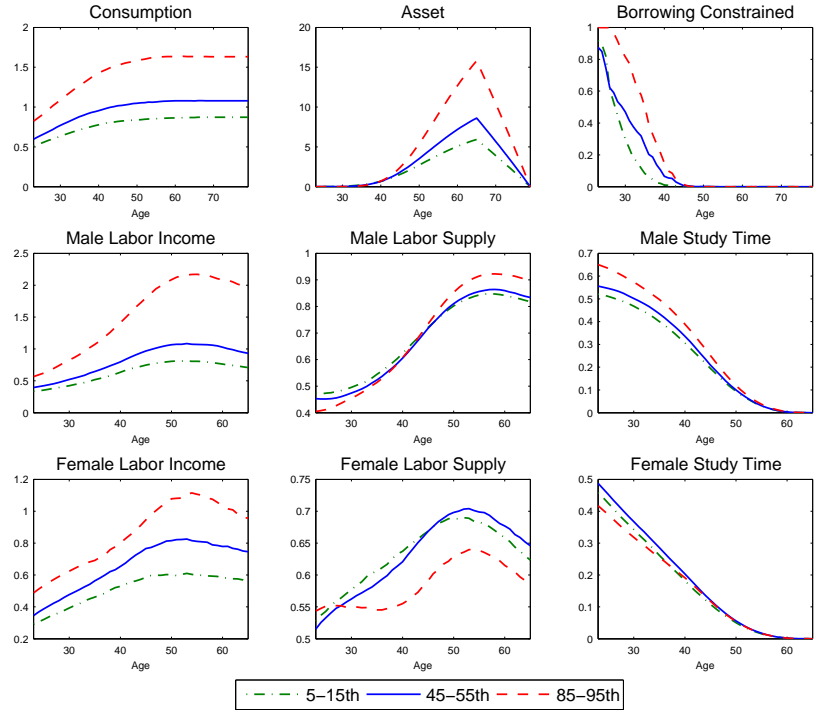


Figure 2.3: Life Cycles in the Model by Ability (2010-2012)

two motives for households to save: the precautionary savings against future idiosyncratic risk and the savings for consumption after retirement. On average, households have almost no savings until their middle 30s and reach the highest wealth level at their retirement, which is common in life-cycle models. As households earn more income and accumulate savings over the life cycle, the share of borrowing-constrained households declines with age to almost zero after age 40.

The male labor income grows with age and reaches its peak around age 55. The male labor supply also grows with age, but the magnitude of the rise is smaller. The difference is the result of increasing labor productivity due to human capital accumulation, which is evident in the life cycle profiles of study time. Over half of the non-leisure time is devoted to human capital investment for the young male earners, and this effort declines over the life cycle as the benefit of human capital investment decreases relative to its cost, which is measured by the current earnings lost due to study. Near retirement, the benefit

of additional human capital investment is very low since little time is left to collect it, whereas the cost is higher because the current wage is high due to previous human capital accumulation. Consequently, human capital investment is almost zero near retirement, and wage declines due to the negative mean of human capital shocks capturing the depreciation of human capital. This is the reason for the declining labor supply and labor income shortly before retirement. Compared with males, the rise of female labor income and labor supply are less significant over the life cycle, and a smaller share of the non-leisure time is devoted to human capital investment. This is because females are disadvantaged in production based on the calibrated technology parameters,<sup>30</sup> and therefore the benefit of human capital investment is lower for females.

Depending on the learning ability, household life cycle profiles are quite different. For males, the high ability earners study more and work less than the low ability earners when they are young because the return to human capital investment is higher for them. Therefore, they have a much steeper rising labor income profile over the life cycle. Also, because the initial labor productivity is positively correlated with an earner's learning ability, and the existence of tight borrowing constraints dictates households to finance their early consumption with contemporaneous income, the labor income of high ability males is still higher than that of the low ability ones in spite of less time worked.<sup>31</sup> As human capital accumulation almost completes after age 50, the wages of high ability males are much higher. Hence, they work longer hours, and the male labor income inequality within the cohort reaches its peak over the life cycle.

For females, the life cycle profiles of study time are no longer monotonic in learning ability. In particular, among the young female earners, both the high and low ability fe-

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<sup>30</sup>This is mainly governed by the combination of  $\theta_2$ ,  $w_{2,1}$ , and the distribution of  $A_2$ .

<sup>31</sup>If there were no borrowing constraints, the high ability earners would borrow to finance their early consumption, devote more time to study, and have lower income than the low ability earners when they are young.



males study less than the middle ability ones. This is partly due to the perfect assortative marriage assumption. In the model, a high ability female is also married with a high ability male who has the same rank in the corresponding ability distribution. Because the expected lifetime income is positively correlated with an earner's ability at equilibrium, this reduces the incentive of the high ability female to increase her future labor income through human capital investment. On the other hand, the low ability females want to increase their labor income, but their return to human capital investment is too low due to their low abilities. While the same argument also works for males, as mentioned earlier, females are disadvantaged in production compared to males, and hence households optimally rely more on the male labor income. This is why the same effect is much weaker for males. For similar reasons, the labor supply of the high ability females is uniformly lower than that of the low ability ones.<sup>32</sup>

Because the high ability households have higher and steeper labor income profile over the life cycle, they have higher consumption, higher peak savings, and are more likely to be borrowing constrained when they are young.

Figure 2.4 plots the same household life cycles for the years 1978-1980. Because the overall productivity of technology is lower than that in the 2010s, the levels of household consumption and savings are both lower. Also, because the return to human capital investment is lower in the 1970s, the male earners study less and work more when they are young. As a result, their labor income life cycle profiles are flatter than those in the 2010s. For the female earners, they are even more disadvantaged in the 1970s than in the 2010s, so their labor income and labor supply decline over the life cycle, and there is almost no human capital accumulation for females. With flatter income profiles over the life cycle, a

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<sup>32</sup>The trough of labor supply for the high ability females around age 40 is because: before age 40, the high ability households are borrowing constrained, and they need the female labor income to increase their early consumption and to allow the male earners to study more; around age 40, most of the high ability households are away from the borrowing constraints, so the high ability females allocate more non-leisure time in study; after age 40, as the wage of high ability females increases, so is their labor supply.

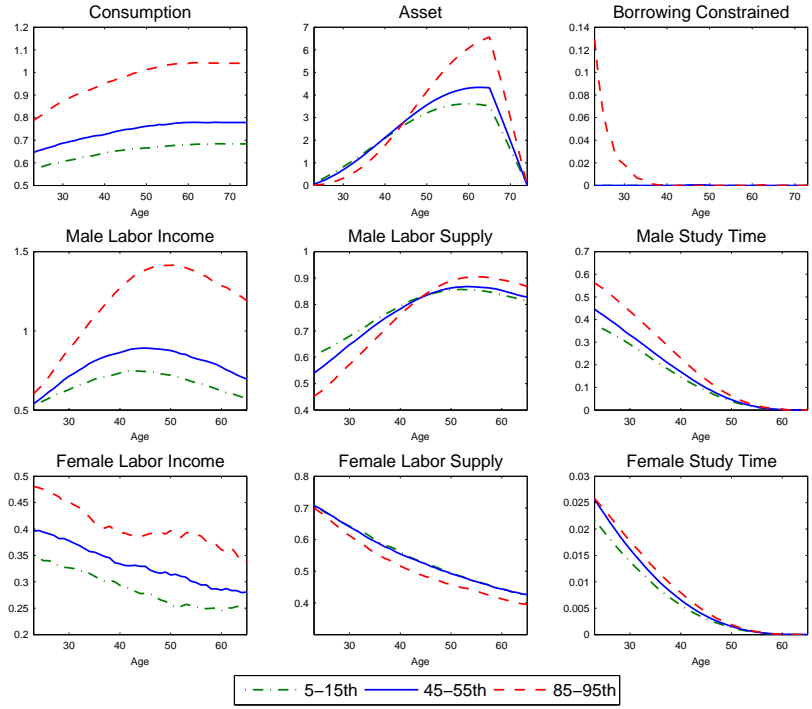


Figure 2.4: Life Cycles in the Model by Ability (1978-1980)

much smaller share of young households are borrowing constrained.

### 2.4.2 Inferred Pareto Weights in the 1970s and 2010s

In Section 2.2.5 and 2.2.6, I have described the Ramsey optimal tax policy problem and the method to recover the Pareto weights implied by the actual income tax policy in that framework. The Pareto weights inferred this way capture the effects of non-economic forces in the determination of income tax policy as the economic forces are already taken into account by the household lifetime utilities in the objective function of the Ramsey problem.

One non-economic force which is widely recognized as a critical determinant for the actual income tax policy is the political influences of various income groups. However, it is extremely hard to identify those directly. Hence I consider the Pareto weights inferred this way as an indirect measure of the political influences and interpret the change in Pareto

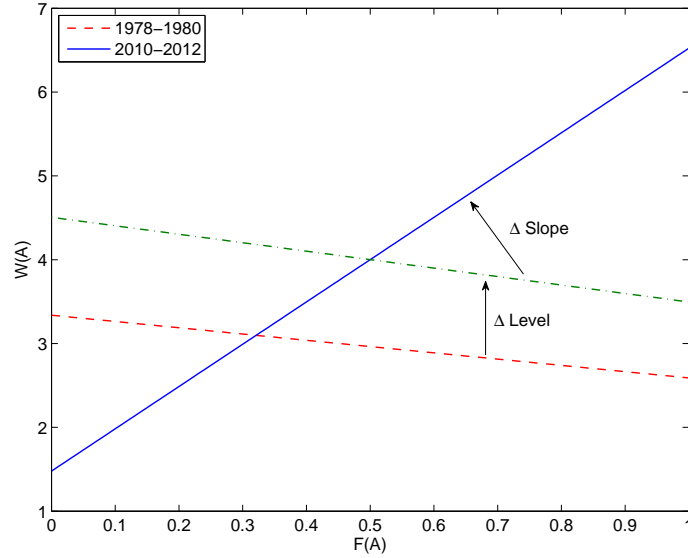


Figure 2.5: Inferred Pareto Weights

weights between the 1970s and 2010s as evidence of changes in political influences. I provide more details including empirical evidences and a political economy model in support of this interpretation in Section 2.5. It is certainly possible that other non-economic forces might also have affected the Pareto weights and the change of them, so my interpretation in this paper is not definite, but rather a good starting point for thinking the non-economic causes of income tax policy change.

Because the calibrated income tax function has only two parameters  $(\tau, \mu)$ , we can identify the Pareto weight function  $W(A)$  up to the first order approximation, i.e., the linear form. I choose  $F(A)$  as the basis function, where  $F(\cdot)$  is the cdf of household learning ability  $A$ . So the functional form assumption of  $W(A)$  is  $W(A) = \xi_0 + \xi_1 F(A)$ , and  $\{\xi_p\}_{p=0}^1$  are coefficients to be inferred from the actual income tax policy. Figure 2.5 reports the Pareto weights inferred from the U.S. income tax policies in the 1970s and 2010s.

For the years 1978-1980, the slope of the Pareto weight function is negative, which means that policymakers value the lifetime utilities of the low ability/income households

more than those of the high ability/income households.<sup>33</sup> The change of the Pareto weight function from the 1970s to 2010s can be decomposed into two steps: a change in level and a change in slope. The change in level is a scale-up of the Pareto weight function while keeping the relative importance of any two households unchanged, i.e, keeping the ratios between the weights at any two ability levels  $\frac{W(A')}{W(A)}$  unchanged. Recall that in the objective function of the Ramsey problem, the weight on government services  $\gamma$  is normalized to be 1 for identification purposes. Therefore, the scale-up of Pareto weights on household private utilities implies a relative decline in the importance of government services, at least as perceived by policymakers. The change in slope is a counterclockwise rotation of the Pareto weight function while keeping the relative importance of household private utilities with respect to government services unchanged, i.e., keeping the area under the weight function  $\int W(A)dF(A)$  unchanged. Due to this change in slope, the Pareto weights assigned to the high ability households are much larger than those at the lower end of the ability distribution in the years 2010-2012. Hence the overall change in Pareto weights between the 1970s and 2010s implies less valued government services and benefits the high ability/income households.

### 2.4.3 Optimal Response of Income Tax Policy to Economic Changes

A key question to answer in this paper is how much of the U.S. income tax policy change since the 1970s can be rationalized as an optimal response of income tax policy to changes in economic fundamentals. To address this question, we need to separate the effects of economic forces and non-economic forces in shaping the income tax policy change we observe. In particular, we can use the Pareto weights inferred from the 1970s income tax policy as a measure of the non-economic forces in the 1970s and then combine them with

<sup>33</sup>Because the expected lifetime income increases with ability, the weights on high/low ability households can also be roughly interpreted as weights on high/low income households.

the economic model calibrated to the 2010s U.S. economy in a Ramsey optimal tax policy problem. The solution to such counterfactual Ramsey problem is the income tax policy which would be chosen in the 2010s if there were *only* changes in economic fundamentals since the 1970s. Hence the difference between the solution to this Ramsey problem and the actual income tax policy in the 1970s gives the optimal response of income tax policy to *only* the economic changes. The remaining change of income tax policy since 1970s is then attributed to the change in Pareto weights by the structure of the Ramsey framework.

The optimal response of income tax policy to economic changes computed using the above method is plotted in Figure 2.6, together with the actual income tax policies in the 1970s and 2010s. In terms of the progressivity of income tax, the elasticity of after-tax income with respect to before-tax income implied by the income tax progressivity parameter  $\mu$  in the tax function is about 0.856 in the 1970s and 0.914 in the 2010s. The optimal response of income tax policy to economic changes implies an elasticity of 0.879. That means the optimal response tax policy is less progressive than the 1970s tax policy but more progressive than the 2010s tax policy and quantitatively account for about 40% of the reduction in progressivity between the 1970s and 2010s. This result is apparent in the right graph of Figure 2.6 where I normalize the tax rates under different tax policies at income level one to eliminate the differences in the level of income tax from the graph.

In terms of the level of income tax, the optimal response of income tax policy is almost the same as the income tax in the 1970s as shown in the left graph of Figure 2.6. The value of the income tax level parameter  $\tau$  in the tax function is about 0.258 in the 1970s, 0.220 in the 2010s, and 0.259 for the optimal response of income tax policy. Therefore, most of the reduction in income tax level since the 1970s is due to the change in Pareto weights, more specifically, the lower weight on government services.

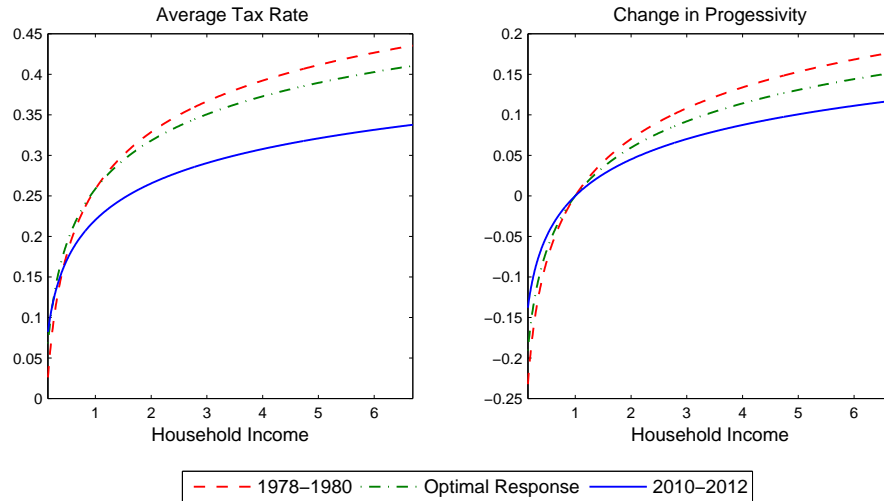


Figure 2.6: Optimal Income Tax Response to Economic Changes

Note: The bounds of household income in the graphs correspond to the cutoff income levels of the first and last tax bracket in 2012. The graph on the right plots the average tax rates under different policies subtracted by the corresponding average tax rates at income level 1, which reflect the levels of income tax  $\tau$  but are not affected by the progressivity parameter  $\mu$ .

#### 2.4.4 Decomposition of Income Tax Policy Change

Several economic changes have occurred since the 1970s as demonstrated by the calibration results of the economic model in Section , including skill-biased technological change, increased female labor productivity, change of idiosyncratic risk, and aging of the U.S. population, etc. To further understand how each economic change contributes to the income tax policy change separately, I conduct a detailed decomposition of the income tax policy change since the 1970s by solving a sequence of counterfactual Ramsey problems.

The exercise starts from the Ramsey problem with the 1970 Pareto weights and the economic model calibrated to the 1970s U.S. economy. By construction, the actual income tax policy in the 1970s is the solution to this problem. Then I introduce economic changes sequentially into the economic model. After all the economic changes are included, I introduce the change in Pareto weights in two steps: first add the change in level and then the change in slope as defined in Section 2.4.2. Whenever the economic model or the

Table 2.5: Decomposition of Income Tax Policy Change

Due to	% in Total Change of Income Tax	
	Progressivity ( $\mu$ )	Level ( $\tau$ )
<i>A. Economic Changes</i>		
Change of Idiosyncratic Risks	-88.5%	67.7%
Female-biased Technological Change	44.8%	28.7%
Skill-biased Technological Change	18.0%	122.0%
Universal Technological Change	-8.1%	-60.6%
Aging of Population	73.5%	-158.5%
Subtotal	39.8%	-0.7%
<i>B. Change in Pareto Weights</i>		
$\Delta$ Level	-58.8%	179.1%
$\Delta$ Slope	119.0%	-78.4%
Subtotal	60.2%	100.7%
Total Change	-0.058	-0.038

Pareto weights are updated, the optimal income tax policy is solved for the corresponding Ramsey problem. The change of optimal income tax policy between two consecutive steps is ascribed to the change introduced between them. The results of this decomposition are reported in Table 2.5.

For the progressivity of income tax, economic changes overall account for 39.8% of reduction in the data, but each economic change contributions differently to this result, both quantitatively and qualitatively. The first economic change introduced is the change of idiosyncratic risk which includes both the initial labor productivity risk and the human capital shocks over the life cycle. The increase of idiosyncratic risk causes a higher income inequality and raises the redistribution/insurance benefit of progressive taxation. As a result, it contributes negatively to the reduction in progressivity and accounts for -88.5% of the change in the data. The female-biased technological change increases female labor productivity, which is captured in the model by the increase of  $\theta_2$ . Because female labor

supply is more elastic than male labor supply, this change requires the optimal income tax to be less progressive and can account for 44.8% of the reduction in progressivity. The skill-biased technological change increases the return to human capital accumulation  $R_H^i$  in the model and benefits the high ability earners more than the low ability ones. This induces a higher income inequality. However, it also increases the efficiency cost of progressive taxation, which discourages human capital investment. The higher income inequality requires a more progressive income tax for redistribution, whereas the larger efficiency cost demands the tax policy to move in the opposite direction. In the end, my quantitative result suggests that the efficiency cost channel dominates the redistribution channel, and overall the skill-biased technological change accounts for 18.0% of the reduction in progressivity. The universal technological change represents the decline of overall labor productivity, i.e., the declining wage of effective labor  $w_e$  in the model. This change has a relatively small effect on progressivity because it affects all earners in a similar way. Finally, the aging of the U.S. population, i.e., larger values of  $T_R$  and social security benefits  $b$  in the model, increases the age dependency ratio, which means more tax revenues need to be collected from the working age population to finance the rising demand for social security benefits and government services. This change results in a less progressive optimal income tax to boost tax revenues and explains 73.5% of the reduction in progressivity.

The change in Pareto weights accounts for the rest 60.2% of reduction in progressivity, more specifically, the change in slope of the Pareto weight function  $W(A)$ . The change in level of the Pareto weight function reduces the importance of government services. Therefore, it lowers the demand for tax revenues and actually requires a more progressive income tax to be adopted. However, this effect of change in level on progressivity is completely offset and reversed by the change in slope of the Pareto weight function, which benefits the high ability/income households and produces a much less progressive income tax policy in the end.



For the level of income tax, the economic changes which raise the total income lead to lower level of income tax, such as the change of idiosyncratic risks and female-biased and skill-biased technological changes, whereas the economic changes which reduce the total income or raise the demand for tax revenues increase the level of income tax, such as the universal technological change and aging of population. In spite of the significant impacts of each economic change, their effects counteract each other, and hence the comprehensive effect of economic changes on the income tax level is quite small. On the other hand, the change in level of Pareto weights is responsible for most of the reduction in level of income tax since the 1970s.

## **2.4.5 Income Inequality, Growth, and Income Tax Policy**

Through the lens of the economic model calibrated to match the economic changes between the 1970s and 2010s, we can ask how each economic change contributes to the rising income inequality and income growth since the 1970s, and what are the roles of the income tax policy in those changes. To answer this question, I conduct a decomposition for the change of labor income inequality and mean labor income similar to the one for the change of income tax policy in Section 2.4.4. Starting with the economic model calibrated to the 1970s U.S. economy, economic changes are introduced sequentially into the model, and the income tax change is added in the last step of this exercise. The decomposition results are reported in Table 2.6.

The income inequality is measured by the ratio between the 90th percentile and 50th percentile of the cross-sectional labor income distribution in the model. I report the decomposition results with this measure of inequality for both the male labor income and household labor income.<sup>34</sup> The decomposition shows that the increase of initial productiv-

<sup>34</sup>The change of this ratio from the years 1978-1980 to 2010-2012 in the CPS data is 0.684 for working males and 0.312 for married working couples. Although the model is not directly calibrated to match these ratios, we can see it matches these rising inequality patterns in the data reasonably well.

Table 2.6: Sources of Rising Income Inequality and Income Growth

Due to Change of	% in Total Change of			
	p90-p50 Income Ratio		Mean Labor Income	
	Male	Household	Male	Female
Initial Productivity Risk	36.4%	65.5%	67.6%	2.4%
Human Capital Shocks	5.9%	-0.5%	5.1%	-12.4%
Female-biased Technological Change	6.3%	-18.6%	-31.8%	57.0%
Skill-biased Technological Change	53.7%	44.0%	172.9%	70.9%
Universal Technological Change	-3.0%	0.3%	-128.1%	-34.8%
Other Economic Factors	-3.7%	5.3%	-5.0%	2.7%
Income Tax	4.4%	3.9%	19.4%	14.2%
Total Change	0.670	0.279	\$8182	\$18284

ity risk and skill-biased technological change are the two main causes of the rising inequality of both the male labor income and household labor income since the 1970s. Notably, female-biased technological change increases the male labor income inequality but reduces the household labor income inequality. This is a result of behavior interaction between the male and female earners within households. As female labor productivity increases, the idiosyncratic shocks to females become more important to household income. And hence, the male earners have to adjust their behaviors more in response to these shocks, such as labor supply and human capital investment, to smooth household income. That is why the male labor income inequality increases. However, at the same time, the higher female labor productivity increases the ability of female earners to provide insurance against the idiosyncratic shocks to males. As a result, it reduces the dispersion of income at household level. The change of income tax policy since the 1970s increases the income inequality due to the less progressive tax schedule, but the contribution is much smaller relative to those of the two main economic causes.

The income growth is measured by the change of mean labor income of males and females. For males, the most important source of income growth is skill-biased technolog-

ical change which increases the return to human capital investment, but a significant part of its effect is offset by the universal technological change which reduces the labor productivity for all earners. Female-biased technological change which increases female labor productivity is an additional contributing factor to the growth of female labor income, but it reduces the male labor income at the same time because households optimally shift a part of the burden of earning income to the female earners. The change of income tax policy since the 1970s plays a more significant role in the income growth than in the rising income inequality, and it accounts for 19.4% and 14.2% of the growth of mean labor income of males and females. In terms of 2012 dollars, the growth of mean labor income due to the change of income tax policy is about \$1587 and \$2596 per year for males and females, respectively.

Overall, economic changes are the main causes of both the rising income inequality and income growth since the 1970s. The change of income tax policy since the 1970s contributes to both of them positively but has a larger effect on income growth. This indicates the classic trade-off between equity and efficiency for the income tax policy.

#### **2.4.6 Optimal Income Tax with Utilitarian Weights**

Following the convention of optimal tax policy literature, I also compute the optimal income tax policy for the early 2010s U.S. economy with utilitarian Pareto weights as the normative criterion in the Ramsey problem. One important choice to make in this exercise is how much the government services should be valued relative to household private utilities. Since the weight on government services is normalized to be one in the objective function of the Ramsey problem, this is equivalent to choosing the constant value of utilitarian weights on household lifetime utilities. I consider two alternative values for this choice, and for each of them, the relative importance of government services corresponds

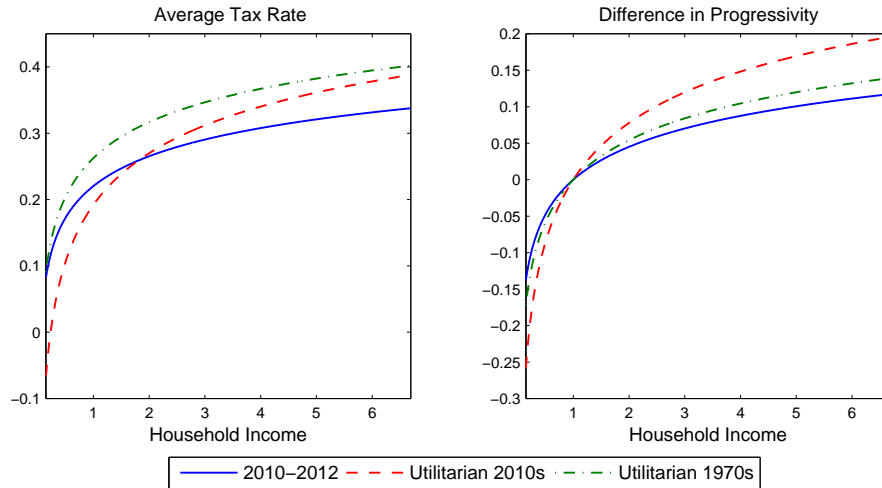


Figure 2.7: Optimal Income Tax with Utilitarian Weights

Note: “Utilitarian 2010s” and “Utilitarian 1970s” denote the optimal income tax policy for the 2010s U.S. economy with utilitarian weights and relative importance of government services implied by the 2010s and 1970s income tax policies. The bounds of household income in the graphs correspond to the cutoff income levels of the first and last tax bracket in 2012.

to that implied by the income tax policy in the 1970s and 2010s, respectively.<sup>35</sup>

Figure 2.7 plots the optimal and the actual income tax policies for the early 2010s. As the graphs show, both the progressivity and level of the optimal income tax depend on the relative importance of government services. If government services are less important as implied by the actual income tax policy in the 2010s, the optimal income tax should be much more progressive and lower in level. The elasticity of after-tax income with respect to before-tax income should be 0.854 relative to the value of 0.914 for the actual income tax in the 2010s, and the level of income tax as measured by  $\tau$  should be 0.192 relative to the value of 0.220 in reality. However, if government services are more important as implied by the actual income tax policy in the 1970s, the progressivity of the optimal income tax would reduce and imply an elasticity of 0.890, closer to the actual income tax in the 2010s. And the level of the optimal income tax would rise significantly to  $\tau = 0.263$ .

<sup>35</sup>More specifically, the two levels of utilitarian weights are:  $W_U^{2010s} = \int W^{2010s}(A)dF(A)$  and  $W_U^{1970s} = \int W^{1970s}(A)dF(A)$ . The 1970s income tax policy implies a more valued government services than the 2010s income tax policy.

One interesting takeaway from this exercise is that if government services become more important or the government wants to collect more tax revenues, the economically optimal response of income tax policy is to lower the progressivity and raise the overall level of income tax, assuming the preferences of policymakers over different households remain the same. From this point of view, the recent increase of marginal income tax rate for the top income bracket in the year 2013 is more likely to be a result of “rebalancing” the Pareto weights rather than rebalancing the government budget and collecting more tax revenues.

## 2.5 Explanations for the Change in Pareto Weights

The quantitative results in Section 2.4 show that the change of income tax policy in the U.S. since the late 1970s implies a change in Pareto weights benefiting the high ability/income households, which contributes significantly to the reduction in progressivity of income tax we observe. In this section, I provide potential explanations for this change in Pareto weights from a political economy point of view. In particular, I discuss two possible causes of this change: (1) the lower cost of conveying information to swing voters due to information technology improvement; (2) the rising inequality of voter turnout among different socioeconomic groups. I first present empirical evidences in support for these explanations and discuss the intuitions. Then in a stylized probabilistic voting model with political contributions, I derive a closed-form expression for the Pareto weight function in the Ramsey framework and show that the two causes proposed can indeed induce a change in Pareto weights benefiting the high ability/income households, consistent with what the change of income tax policy implies in Section 2.4.

## 2.5.1 Empirical Evidences

### Money in Political Campaigns

It is not a secret that money plays an important role in political elections. In the 2012 presidential campaign, Barack Obama and Mitt Romney each spent around 1 billion dollars, which is arguably enough to send a person to the moon. Most of the money was spent on media and other forms of political persuasion to potential voters. Previous studies have shown that the political information, whether biased or not, delivered by media has real effects on voting behaviors. For example, DellaVigna and Kaplan (2007) find that the introduction of Fox News between October 1996 and November 2000 convinced 3 to 28 percent of its viewers to vote Republican, depending on the audience measure; Ladd and Lenz (2009) estimate that the endorsement switch to the Labour Party by several prominent British newspapers before the 1997 United Kingdom general election persuaded 10 to 25 percent of their readers to vote for Labour, depending on the statistical approach.

There is evidence that the importance of money in the U.S. elections has increased. Figure 2.8 plots the normalized real average campaign expenditures per candidate for the U.S. House of Representatives and Senate since 1974.<sup>36</sup> It is apparent that campaign expenditures have grown a lot and faster than GDP. Since politicians can only use their campaign funds in elections to improve their chances of winning, this sharp rise of campaign expenditures relative to GDP implies that money might have become more effective for the purpose of gaining votes.

Why money has become more important in political campaigns? One possible reason is the lower cost of conveying information to swing voters due to information technology improvement. It is obvious that the cost of passing through information to voters is much lower today than in the 1970s due to the expansions of television and telephone networks

<sup>36</sup>Source: Campaign Finance Institute analysis of Federal Election Commission data.

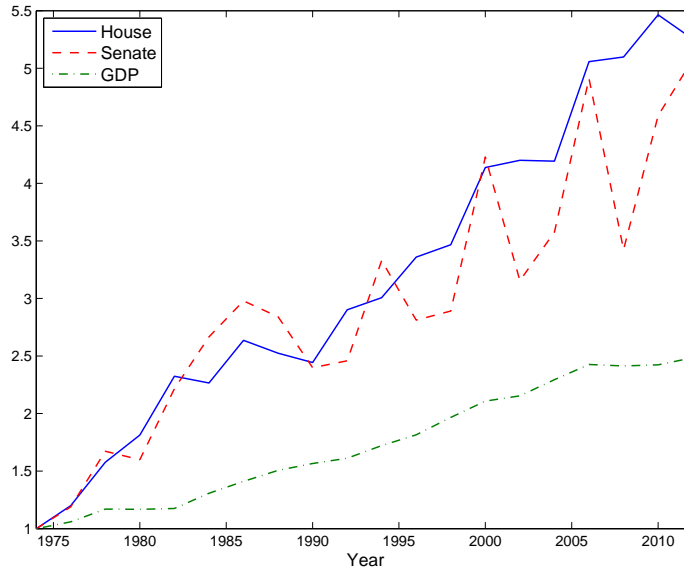


Figure 2.8: Average Campaign Expenditures per Candidate

Note: All the series are deflated using Consumer Price Index Research Series (CPI-U-RS). The starting values of all the series at the year 1974 are normalized to be 1, and the series plotted are the actual values divided by their 1974 values.

and most recently the internet. The lower transportation costs today also make it easier for politicians and voters to gather in person more often. Political campaigns are all about conveying information to voters and persuading them to vote accordingly. The advantage of spending one more dollar than the opponents depends on how many additional information flows to voters can be generated with this amount of money. When the cost of information flows is high, the incentive of politicians to collect and spend more money in such activity is low. On the other hand, when that cost is lower, politicians may devote more efforts in fund-raising activities and spend larger amount of money in all kinds of media to improve their chances of winning as we observe nowadays.

The increased demand for campaign funds may induce politicians to propose policies more favorable to the high-income households because the high-income households are typically more willing and able to donate more to their preferred politicians. This could potentially explain the change in Pareto weights benefiting the high-income households

inferred from the change of income tax policy in Section 2.4.

### **Rising Inequality of Voter Turnout**

A basic fact about political elections is that not all people participate in voting. For example, only 56.5% of voting age population voted in the year 2012. And the participation rate, i.e, voter turnout, varies a lot among populations of different socioeconomic groups. Data suggest that voter turnout increases with income, educational attainment and age. Since the 1970s, there are evidences that such inequality of voter turnout among socioeconomic groups has increased. Figure 2.9 plots voter turnout for population groups with different educational attainments (hence different learning ability) since the mid-1970s.<sup>37</sup> Voter turnout has declined over time for all the groups, but the reduction is more significant for the groups with lower educational attainments. Consequently, the shares of votes from the low education groups in elections have declined, whereas the opposite is true for the high education groups. Since politicians care about winning elections, they certainly should have responded to this change in the composition of voters by adjusting their policies towards the high education/income households. Similar patterns exist for population groups with different income levels, and I refer the reader to Freeman (2003) for more detailed information and discussions.

### **2.5.2 A Probabilistic Voting Model with Political Contributions**

Motivated by the empirical evidences and intuitions presented in Section 2.5.1, I build a stylized probabilistic voting model with political contributions to formalize the argument and derive a closed-form expression for the Pareto weight function in the Ramsey framework, which shows explicitly how different factors affect the Pareto weights. Unlike the quantitative life-cycle model in Section 2.2, which is used to match data and deliver re-

<sup>37</sup>Source: U.S. Census Bureau, Current Population Survey, Voting and Registration data.



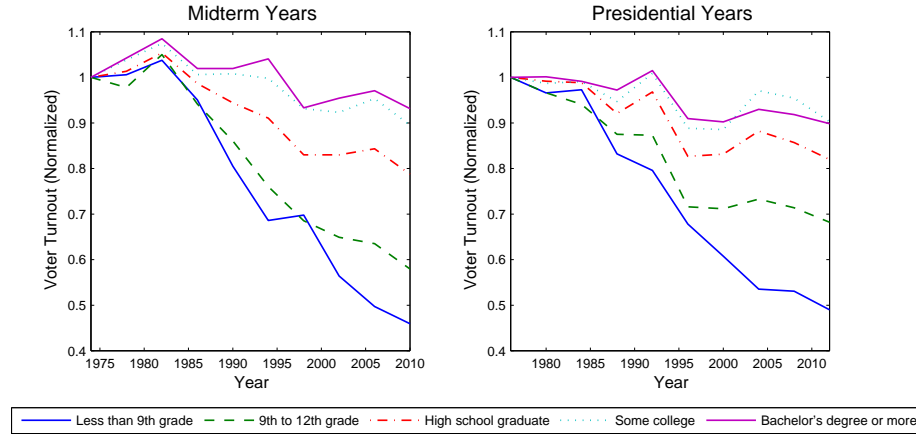


Figure 2.9: Voter Turnout by Educational Attainments

Note: The starting values of all the series at the year 1974 (1976) are normalized to be 1, and the series plotted are the actual values divided by their 1974 (1976) values.

liable numerical results, the probabilistic voting model here is mainly used to illustrate ideas and help us understand the determination of Pareto weights from a political economy point of view. Hence the economic side of the model is simplified to avoid unnecessary complications. The notations in this section are independent from other parts of this paper.

Consider an economy populated by a measure one of infinitely-lived households with heterogeneous ability  $A$ . The distribution of ability is governed by the cdf  $F(A)$ . The before-tax income of a household  $y_A$  is exogenous and increases with its ability level. Households are not allowed to save or borrow for simplicity. Let  $\tau$  denote the income tax policy,<sup>38</sup> then household consumption is simply equal to the after-tax income, i.e.,  $c_A(\tau) = y_A - T(y_A, \tau)$ , where  $T(\cdot, \tau)$  is the income tax function governed by  $\tau$ . The instantaneous utility function is  $u(\cdot)$ .

Suppose for time  $t \leq T$ , the policy is  $\tau^0$ , and an election is organized at the end of time  $T$ . In this election, two candidates (or parties),  $a$  and  $b$ , propose future policy  $\tau^a$  and  $\tau^b$  for time  $t \geq T + 1$ , and have full commitment power. The objective of each candidate is to

<sup>38</sup>In general,  $\tau$  can be a vector, but for notation's sake, I write it as a scalar when deriving the formulas. It is straightforward to extend them to the vector case.

maximize its own number of votes received.<sup>39</sup> Each household has the right of exactly one vote, and voter turnout is  $\pi_v(A)$  for the ability  $A$  households.<sup>40</sup>

Conditional on voting, an ability  $A$  household can either be a determined voter with probability  $\pi_d(A)$  or a swing voter with probability  $\pi_s(A) = 1 - \pi_d(A)$ . The determined voters vote according to their discounted utilities from time  $t \geq T + 1$  under the policies proposed by candidates,  $V_A(\tau^a)$  and  $V_A(\tau^b)$ . There is also an idiosyncratic component  $\varepsilon_A$  reflecting the heterogeneous tastes among the ability  $A$  voters with respect to other characteristics of candidates.  $\varepsilon_A$  is i.i.d across voters and follows a uniform distribution governed by  $\phi_A$ , i.e.,  $\varepsilon_A \sim Unif(-\frac{1}{2\phi_A}, \frac{1}{2\phi_A})$ . The probability of an ability  $A$  determined voter to vote for candidate  $a$  is then

$$\begin{aligned} \Pr\{\text{vote for } a|(d, A)\} &= \Pr\{V_A(\tau^a) - V_A(\tau^b) > \varepsilon_A\} \\ &= \phi_A[V_A(\tau^a) - V_A(\tau^b)] + \frac{1}{2} \equiv P_A^a(\tau^a, \tau^b) \end{aligned}$$

where  $d$  represents the type “determined”.<sup>41</sup>

The voting behaviors of swing voters are similar to those of determined voters, but their preferences over the two candidates can be affected by the amounts of information they receive from each candidate. The information flows from a specific candidate to a swing voter increase the probability of the swing voter to vote for this candidate and reduces the same amount of that for the other candidate.

To finance the money costs of information flows to swing voters, candidates need to raise campaign funds from the group of determined voters who support them. In particular,

<sup>39</sup>This case is like two parties fighting over the number of seats in congress. If we assume the candidates maximize their probabilities to win, i.e., probabilities of receiving more than a half of the total votes, instead of maximizing the number of votes received, the qualitative results would be similar in the end, but the notations would become more complicated.

<sup>40</sup>Voter turnout is defined as the percentage of voting age population who actually vote in the election.

<sup>41</sup>The value of  $V_A(\tau^a) - V_A(\tau^b)$  is assumed to be within the support of the uniform distribution when calculating the probability. This is not an issue because I focus on the symmetric Nash equilibrium later at which  $V_A(\tau^a) - V_A(\tau^b)$  is 0.

a candidate can devote effort  $e$  in fund-raising activities, and encourages its own determined voters to donate a part of their current consumption as political contributions to the candidate. On the other side, determined voters enjoy providing support to the candidate they would vote for, and their utility function is assumed to be

$$u(c) = \max_{(c_p, c_d)} u\left(\left(\frac{c_p}{1 - \chi(e)}\right)^{1-\chi(e)} \left(\frac{c_d}{\chi(e)}\right)^{\chi(e)}\right)$$

s.t.

$$c_p + c_d = c, c_p \geq 0, c_d \geq 0$$

where  $c$  is the total consumption,  $c_p$  is the private consumption, and  $c_d$  is the political contribution;  $\chi(e) \in (0, 1)$  controls the weight of political contribution in the determined voter's utility, and it satisfies  $\chi'(0) = +\infty$ ,  $\chi'(1) = 0$ , and  $\chi''(\cdot) < 0$ . With this specification of utility function, a determined voter always donates  $\chi(e)$  share of its total consumption  $c$ , but its total utility level is not affected by the effort  $e$ .<sup>42</sup>

Each candidate is endowed with total one unit of effort, and hence only  $1 - e$  unit of effort can be devoted to non-fund-raising activities, which increase the probability for a swing voter to vote for this candidate by  $B(1 - e)$  and reduce the same amount for the other candidate. The function  $B(\cdot)$  satisfies  $B'(0) = +\infty$ ,  $B'(1) = 0$ , and  $B''(\cdot) < 0$ .

Therefore, the probability of an ability  $A$  swing voter to vote for candidate  $a$  is

$$\Pr\{\text{vote for } a | (s, A)\} = P_A^a(\tau^a, \tau^b) + z(n^a - n^b) + B(1 - e^a) - B(1 - e^b)$$

where  $s$  represents the type "swing";  $z$  is the effectiveness of information flows in persuading the swing voter;  $n^a$  and  $n^b$  are the amounts of information flows delivered to this

<sup>42</sup>This setting of utility function is to make sure that the voting decisions of determined voters are not affected by the fund-raising efforts of candidates.

swing voter by candidate  $a$  and  $b$ ;  $e^a$  and  $e^b$  are the levels of effort devoted to fund-raising activities by candidate  $a$  and  $b$ .

Given candidate  $b$ 's choice of  $(\tau^b, e^b)$ , the optimal campaign strategy problem of candidate  $a$  is

$$\begin{aligned} \max_{(\tau^a, e^a, n^a, n^b)} & \int P_A^a(\tau^a, \tau^b) \pi_d(A) \pi_v(A) dF(A) \\ & + \int [P_A^a(\tau^a, \tau^b) + z(n^a - n^b) + B(1 - e^a) - B(1 - e^b)] \pi_s(A) \pi_v(A) dF(A) \end{aligned}$$

s.t.

$$\begin{aligned} pn^a \int \pi_s(A) \pi_v(A) dF(A) &= \int \chi(e^a) c_A(\tau^0) P_A^a(\tau^a, \tau^b) \pi_d(A) \pi_v(A) dF(A) \\ pn^b \int \pi_s(A) \pi_v(A) dF(A) &= \int \chi(e^b) c_A(\tau^0) [1 - P_A^a(\tau^a, \tau^b)] \pi_d(A) \pi_v(A) dF(A) \\ 0 \leq e^a \leq 1, \tau^a \in \Gamma, n^a \geq 0, n^b \geq 0 \end{aligned}$$

where  $p$  is the price of information flows, and  $\Gamma$  is the feasible set of policy  $\tau$ .<sup>43</sup> The first row of the objective function corresponds to the votes for candidate  $a$  from determined voters; the second row corresponds to the votes for candidate  $a$  from swing voters. The choice variables  $n^a$  and  $n^b$  in the objective function can be substituted out using the budget constraints, and the first order conditions of candidate  $a$ 's problem are then:

$$\tau^a : \int \phi_A \left\{ 1 + \frac{z}{p} [\chi(e^a) + \chi(e^b)] c_A(\tau^0) \pi_d(A) \right\} V_A'(\tau^a) \pi_v(A) dF(A) = 0; \quad (2.5.1)$$

$$e^a : \chi'(e^a) \frac{z}{p} \int c_A(\tau^0) P_A^a(\tau^a, \tau^b) \pi_d(A) \pi_v(A) dF(A) = B'(1 - e^a) \int \pi_s(A) \pi_v(A) dF(A). \quad (2.5.2)$$

<sup>43</sup>The reason why  $n^b$  is also a choice variable and the budget constraint of candidate  $b$  is also a constraint for candidate  $a$  is because the choice of  $\tau^a$  affects the budget constraint of candidate  $b$  and hence  $n^b$ , which should be taken into consideration by candidate  $a$ . In other words, the strategy of each candidate is only  $(\tau, e)$ , and  $(n^a, n^b)$  are endogenously determined by the budget constraints.

Since candidate  $a$  and  $b$  are identical except for their names, I focus on the symmetric pure strategy Nash equilibrium at which both candidates chose the same optimal strategy, i.e.,  $\tau^{a*} = \tau^{b*} = \tau^*$  and  $e^{a*} = e^{b*} = e^*$ .

Define the function  $W(A)$  as

$$W(A) \equiv \phi_A \left[ 1 + 2 \frac{\tilde{z}}{p} \chi(e^*) c_A(\tau^0) \pi_d(A) \right] \pi_v(A). \quad (2.5.3)$$

Then Equation (2.5.1) at the symmetric Nash equilibrium becomes

$$\int W(A) V'_A(\tau^*) dF(A) = 0,$$

which is the same as the first order condition of a Ramsey optimal policy problem with the Pareto weight function  $W(A)$ . Hence, the equilibrium policy  $\tau^*$  from the political economy model is also the solution to the Ramsey problem with Pareto weight function  $W(A)$ ,<sup>44</sup> i.e.,

$$\tau^* = \arg \max_{\tau \in \Gamma} \int W(A) V_A(\tau) dF(A).$$

This justifies the use of the Ramsey framework as a parsimonious way to model the actual policy-making process in the quantitative study of this paper.

Equation (2.5.3) offers a closed-form expression for the Pareto weights in the Ramsey framework, and hence it allows us to link changes of different factors into changes of Pareto weights. Among all the factors, those corresponding to the previous discussion in Section 2.5.1 are the price of information flows  $p$  and the ratios  $\frac{\pi_v(A_1)}{\pi_v(A_2)}$  between different ability levels. Proposition 1 states how the Pareto weight function is related to these two factors.<sup>45</sup>

**Proposition 1** *Let  $A_1$  and  $A_2$ ,  $A_1 > A_2$ , denote two ability levels, then under the assump-*

<sup>44</sup>Suppose the first order condition is both necessary and sufficient.

<sup>45</sup>The proof of Proposition 1 is straightforward given the expression for Pareto weights in Equation (2.5.3) and the functional form assumptions on  $\chi(\cdot)$  and  $B(\cdot)$ .

tions made in the model, the ratio of Pareto weights between these two ability levels,  $\frac{W(A_1)}{W(A_2)}$ , is

1. decreasing with  $p$  if  $c_{A_1}(\tau^0)\pi_d(A_1) > c_{A_2}(\tau^0)\pi_d(A_2)$ ;
2. increasing with the ratio  $\frac{\pi_v(A_1)}{\pi_v(A_2)}$  if  $e^*$  is fixed.

Mapping the results in Proposition 1 to the discussions in Section 2.5.1: (1) A decrease of  $p$  lowers the cost of conveying information to swing voters, and hence it increases the effectiveness of money in gaining votes for candidates. The model predicts that this change increases the Pareto weights of the high ability households as long as the high ability households have higher  $c_A(\tau^0)\pi_d(A)$  than the low ability households, which is likely to be the case in reality.<sup>46</sup> The model also implies that more efforts are devoted by candidates to the fund-raising activities when  $p$  is lower, and therefore the total political contributions rise relative to the size of the economy, which is consistent with the empirical evidence on campaign expenditures in Figure 2.8. (2) In the model, the ratio  $\frac{\pi_v(A_1)}{\pi_v(A_2)}$  measures the inequality in voter turnout between the high and low ability households. Figure 2.9 shows that this ratio has increased in the data. In response to such change, the model predicts an increase of  $\frac{W(A_1)}{W(A_2)}$ , i.e., the Pareto weights on the high ability households would increase relative to those on the low ability households, if the fund-raising effort remains the same.<sup>47</sup> Therefore, the political economy model confirms that the two proposed explanations for the change in Pareto weights, the lower cost of conveying information to swing voters and the rising inequality of voter turnout, would indeed induce a change in Pareto weights benefiting the high ability/income households, consistent with what is inferred from the change of

<sup>46</sup>The high ability (income) households clearly should have higher consumption. The share of determined voters  $\pi_d(A)$  is harder to measure directly, but the high ability households typically participate more in political activities and receive better education, so they should be less affected by the media, i.e., be more determined and believe in their own knowledge and judgement.

<sup>47</sup>If the fund-raising effort is allowed to respond to the changing inequality of voter turnout, the final result depends on the quantitative properties of the model such as the exact distributions of voter turnout, household consumption and share of determined voters, etc.

income tax policy in the quantitative study.

## 2.6 Conclusions

In this paper, I examined the causes of the less progressive income tax in the U.S. since the 1970s in the Ramsey optimal tax policy framework. Using a quantitative life-cycle model with heterogeneous households calibrated to match the U.S. economy in the late 1970s and early 2010s, I showed that changes in economic fundamentals alone require a less progressive optimal income tax to be adopted and can account for 40% of the reduction in progressivity we observe. In particular, skill-biased technological change, increased female labor productivity, and the aging U.S. population contribute to this reduction in progressivity, but their effects are partially offset by the increase of idiosyncratic risk. The remaining change of income tax implies a change in Pareto weights in the Ramsey framework. The change in Pareto weights benefits the high ability/income households and lowers the importance of government services. Finally, I proposed two potential explanations for this change in Pareto weights from a political economy point of view: the lower cost of conveying information to swing voters due to information technology improvement and the rising inequality of voter turnout among different socioeconomic groups.

The quantitative life-cycle model in this paper has already shown its success in matching several aspects of the data, but there is still room for improvement such as the inclusion of female extensive margin and an explicit modeling of the marriage and divorce processes. These features are not in the model of this paper due to the limit of computing power. With more computing power available or some simplifying assumptions on other aspects of the model, it might be possible to take those features into account in future work and examine how they affect female and household behaviors such as labor supply and human capital accumulation decisions.

The Ben-Porath style human capital accumulation in the model of this paper represents “on-the-job training”. There is another way of modeling human capital accumulation in the literature, which is “learning-by-doing”. It would be helpful for future work to build a quantitative life-cycle model with “learning-by-doing” human capital accumulation, and examine how different the implications on income tax policy are.

Finally, my quantitative study ascribes a significant part of observed change in income tax policy to a change in Pareto weights benefiting the high-income households. I interpret it as evidence of changing political influences of various income groups and provide my own explanations. My political economy model only considers a one-shot income tax policy change, and hence it does not allow the possible effects of the tax policy change on the distribution of political influence, which could again affect future tax policy. Future work allowing this channel from income tax policy to the distribution of political influence could provide more insight into this issue and help us understand the evolution of income tax policy and the distribution of political influence jointly.



# Bibliography

- Acemoglu, Daron (2002). “Technical Change, Inequality, and the Labor Market”. In: *Journal of Economic Literature* 40.1, pp. 7–72.
- Aiyagari, S. Rao (1994). “Uninsured Idiosyncratic Risk and Aggregate Saving”. In: *The Quarterly Journal of Economics* 109.3, pp. 659–684.
- Badel, Alejandro and Mark Huggett (2014). *Taxing Top Earners: A Human Capital Perspective*. Working Paper.
- Bénabou, Roland (2002). “Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?” In: *Econometrica* 70.2, pp. 481–517.
- Blau, Francine D. and Lawrence M. Kahn (2000). “Gender Differences in Pay”. In: *Journal of Economic Perspectives* 14.4, pp. 75–99.
- Blundell, Richard, Luigi Pistaferri, and Ian Preston (2008). “Consumption Inequality and Partial Insurance”. In: *American Economic Review* 98.5, pp. 1887–1921.
- Blundell, Richard, Luigi Pistaferri, and Itay Saporta-Eksten (2012). *Consumption Inequality and Family Labor Supply*. Working Paper 18445. National Bureau of Economic Research.
- Bourguignon, Francois and Amedeo Spadaro (2012). “Taxbenefit revealed social preferences”. English. In: *The Journal of Economic Inequality* 10.1, pp. 75–108.

- Chang, Bo Hyun, Yongsung Chang, and Sun-Bin Kim (2015). *Pareto Weights in Practice: Income Inequality and Optimal Tax*. Working Paper.
- Conesa, Juan Carlos, Sagiri Kitao, and Dirk Krueger (2009). “Taxing Capital? Not a Bad Idea after All!” In: *American Economic Review* 99.1, pp. 25–48.
- Conesa, Juan Carlos and Dirk Krueger (2006). “On the optimal progressivity of the income tax code”. In: *Journal of Monetary Economics* 53.7, pp. 1425 –1450.
- DellaVigna, Stefano and Ethan Kaplan (2007). “The Fox News Effect: Media Bias and Voting”. In: *The Quarterly Journal of Economics* 122.3, pp. 1187–1234.
- Diamond, Peter A. (1998). “Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Tax Rates”. In: *The American Economic Review* 88.1, pp. 83–95.
- Farhi, Emmanuel and Ivn Werning (2013). “Insurance and Taxation over the Life Cycle”. In: *The Review of Economic Studies* 80.2, pp. 596–635.
- Freeman, Richard B. (2003). *What, Me Vote?* Working Paper 9896. National Bureau of Economic Research.
- Guner, Nezih, Remzi Kaygusuz, and Gustavo Ventura (2012). “Taxation and Household Labour Supply”. In: *The Review of Economic Studies* 79.3, pp. 1113–1149.
- Guner, Nezih, Martin Lopez-Daneri, and Gustavo Ventura (2015). *Heterogeneity and Government Revenues: Higher Taxes at the Top?* Working Paper.
- Güvenen, Fatih and Burhanettin Kuruscu (2010). “A Quantitative Analysis of the Evolution of the U.S. Wage Distribution, 1970-2000”. In: *NBER Macroeconomics Annual 2009, Volume 24*. University of Chicago Press, pp. 227–276.
- Güvenen, Fatih, Burhanettin Kuruscu, and Serdar Ozkan (2014). “Taxation of Human Capital and Wage Inequality: A Cross-Country Analysis”. In: *The Review of Economic Studies* 81.2, pp. 818–850.

- Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante (2010). “The Macroeconomic Implications of Rising Wage Inequality in the United States”. English. In: *Journal of Political Economy* 118.4, pp. 681–722.
- (2014). *Optimal Tax Progressivity: An Analytical Framework*. Working Paper 19899. National Bureau of Economic Research.
- Holter, Hans A., Dirk Krueger, and Serhiy Stepanchuk (2014). *How Does Tax Progressivity and Household Heterogeneity Affect Laffer Curves?* Working Paper 20688. National Bureau of Economic Research.
- Huggett, Mark (1993). “The risk-free rate in heterogeneous-agent incomplete-insurance economies”. In: *Journal of Economic Dynamics and Control* 17.5-6, pp. 953 –969.
- Huggett, Mark, Gustavo Ventura, and Amir Yaron (2011). “Sources of Lifetime Inequality”. In: *American Economic Review* 101.7, pp. 2923–54.
- Kaplan, Greg and Giovanni L. Violante (2010). “How Much Consumption Insurance beyond Self-Insurance?” In: *American Economic Journal: Macroeconomics* 2.4, pp. 53–87.
- Kaymak, Baris and Markus Poschke (2015). *Macroeconomic Implications of Top Income Tax Cuts: 1960 - 2010*. Working Paper.
- Kindermann, Fabian and Dirk Krueger (2014). *High Marginal Tax Rates on the Top 1%? Lessons from a Life Cycle Model with Idiosyncratic Income Risk*. Working Paper 20601. National Bureau of Economic Research.
- Krueger, Dirk and Alexander Ludwig (2013). “Optimal Progressive Labor Income Taxation and Education Subsidies When Education Decisions and Intergenerational Transfers Are Endogenous”. In: *American Economic Review* 103.3, pp. 496–501.
- Ladd, Jonathan McDonald and Gabriel S. Lenz (2009). “Exploiting a Rare Communication Shift to Document the Persuasive Power of the News Media”. In: *American Journal of Political Science* 53.2, pp. 394–410.

- Lockwood, Benjamin B. and Matthew Weinzierl (2014). *Positive and Normative Judgments Implicit in U.S. Tax Policy, and the Costs of Unequal Growth and Recessions*. Working Paper.
- Mirrlees, J. A. (1971). “An Exploration in the Theory of Optimum Income Taxation”. In: *The Review of Economic Studies* 38.2, pp. 175–208.
- Moffitt, Robert A. and Peter Gottschalk (1995). *Trends in the Variances of Permanent and Transitory Earnings in the U.S. and Their Relation to Earnings Mobility*. Boston College Working Papers in Economics 444. Boston College Department of Economics.
- O’Neill, June (2003). “The Gender Gap in Wages, circa 2000”. In: *American Economic Review* 93.2, pp. 309–314.
- Piketty, Thomas and Emmanuel Saez (2003). “Income Inequality in the United States, 1913-1998”. In: *The Quarterly Journal of Economics* 118.1, pp. 1–39.
- Saez, Emmanuel (2001). “Using Elasticities to Derive Optimal Income Tax Rates”. In: *The Review of Economic Studies* 68.1, pp. 205–229.
- Stantcheva, Stefanie (2015). *[REVISED] Optimal Taxation and Human Capital Policies over the Life Cycle*. Working Paper.
- Warnock, Francis E. and Veronica Cacadac Warnock (2009). “International capital flows and U.S. interest rates”. In: *Journal of International Money and Finance* 28.6, pp. 903–919.

## 2.A Income Inequality, Gender Gap, and Tax Policy

This section presents empirical facts mentioned in the introduction of this paper. Figure 2.10 reports the time series of the 10th, 50th, and 90th percentiles of the labor income distribution for males and females since the late 1970s. Most of the rising income inequality is caused by the widening gap in the upper half of the distributions. Figure 2.11 plots the 50th and 90th percentiles of the labor income distribution over the life cycle for males and females in the years 1978-1980 and 2010-2012. The income inequality did not change much for the young earners but increased sharply for the middle-age and old earners. The female life cycle profiles of income have become higher in level and steeper in slope. Figure 2.12 plots the time series of the labor income ratio between females and males in the U.S. economy since the late 1970s. The female labor income used to be only one third of male labor income in the late 1970s, but has risen to about two thirds of male labor income in the early 2010s.

Figure 2.13 reports the federal income tax policy since the year 1979 when Jimmy Carter was the president until the year 2013 under the Barack Obama administration. Overall, federal income tax has become lower in level for almost all income levels, but the tax cuts were larger for high-income households. Even though the marginal tax rate at the very top was increased in 2013, most of the tax schedule remained the same as after the Bush tax cut in 2003, and it is still much less progressive than the tax policy in the 1970s.

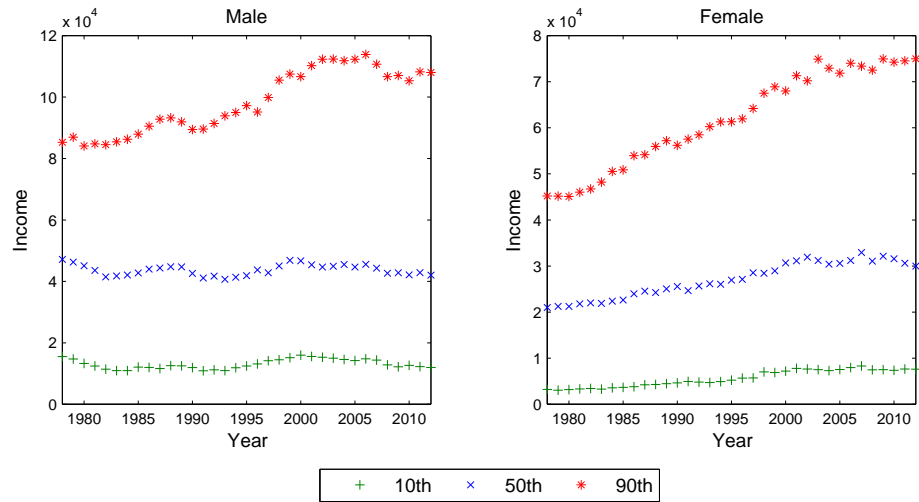


Figure 2.10: Rising Income Inequality since the 1970s

Note: Only labor income is included. The income data are from CPS and have been converted to 2012 dollars.

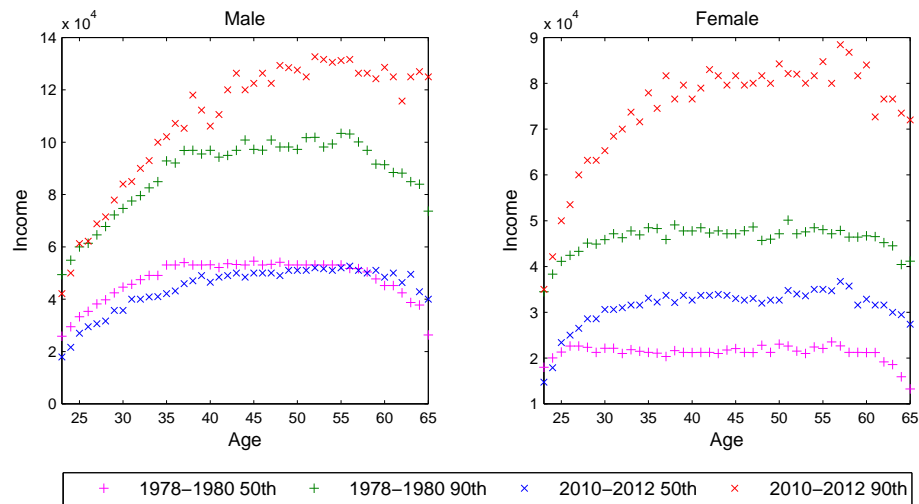


Figure 2.11: Rising Income Inequality over Life Cycle

Note: Only labor income is included. The income data are from CPS and have been converted to 2012 dollars.

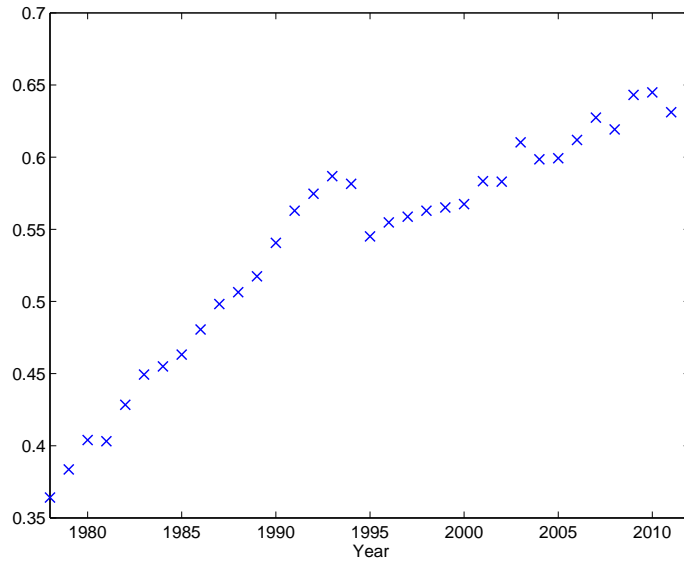


Figure 2.12: Rise of Female-Male Income Ratio

Note: Only labor income is included. The income data are from CPS. The time series plotted is the ratio between total labor income of females and males in each year.

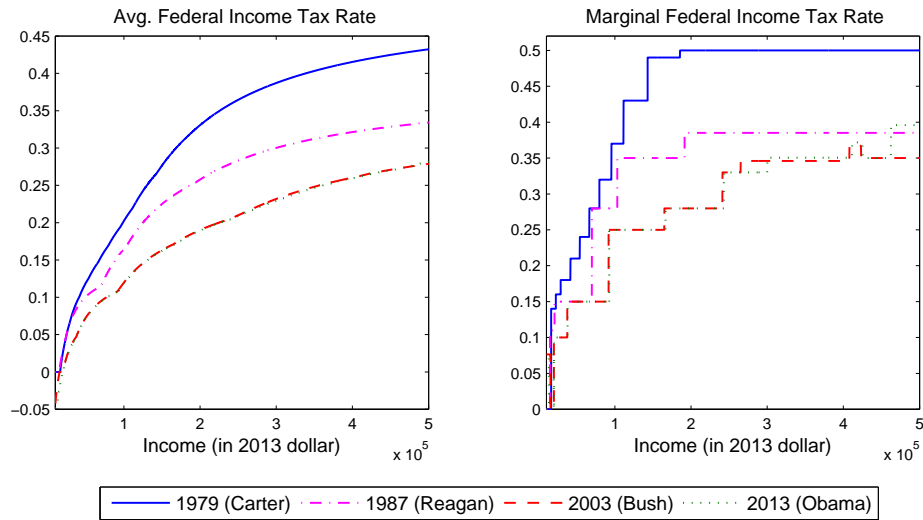


Figure 2.13: The U.S. Income Tax Policy since the 1970s

Note: Income tax policy data are from NBER's TAXSIM program. The tax rates plotted are for married couples filing jointly.

## 2.B Sensitivity Analysis

In this section, I provide sensitivity analysis for the main quantitative results in Section 2.4 and show that the main conclusions of this paper are robust to these variations.

### 2.B.1 Alternative Basis Function for Pareto Weight Function

In Section 2.4.2, I reported the inferred Pareto weight function with  $F(A)$  as the basis function, i.e.,  $W(A) = \xi_0 + \xi_1 F(A)$ . An alternative basis function is simply  $A$ , i.e.,  $W(A) = \xi_0 + \xi_1 A$ . The inferred Pareto weight function with this specification is reported in Figure 2.14. We can see the change in Pareto weights between the 1970s and 2010s with this specification is similar to the benchmark case in the main text. The Pareto weights in the 2010s are higher in level, i.e., the government services are less important, and the change in Pareto weights from the 1970s benefits the high ability/income households.

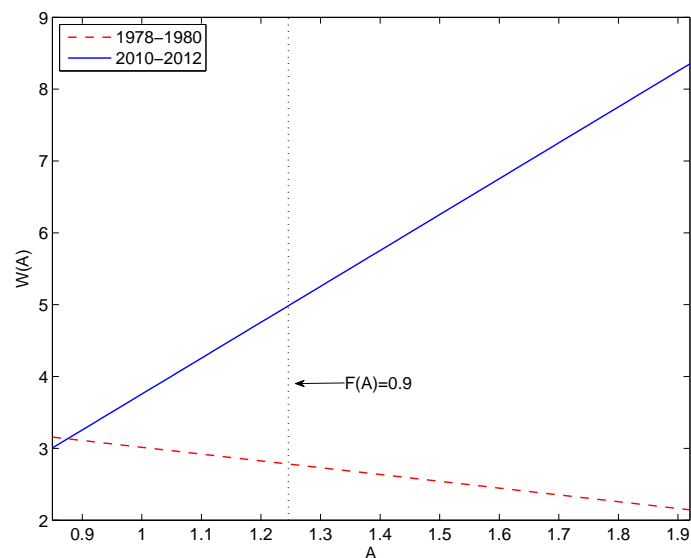


Figure 2.14: Inferred Pareto Weights [ $W(A) = \xi_0 + \xi_1 A$ ]

Note: The lower and upper bounds of ability in the graph correspond to the 1st and 99th percentiles of the ability distribution. The vertical line represents the 90th percentile of the ability distribution.



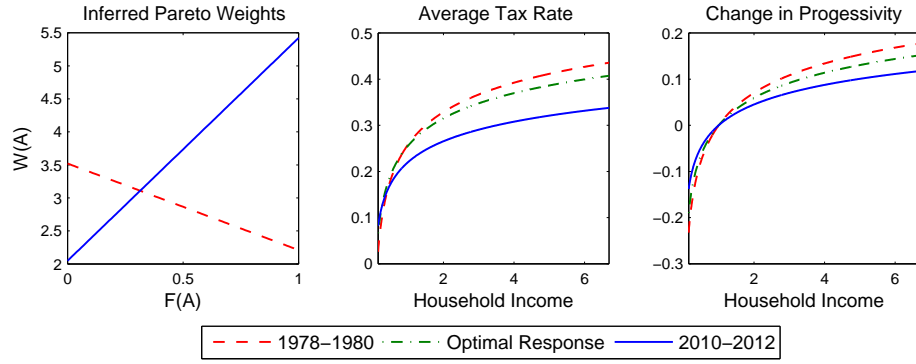


Figure 2.15: Quantitative Results with Alternative Government Budget Constraint

## 2.B.2 Alternative Specifications of the Ramsey Problem

### Alternative Government Budget Constraint

In the benchmark case, the government is assumed to balance its budget period-by-period. An alternative specification is to assume the government to balance its budget cohort-by-cohort:

$$\sum_{j=1}^T \frac{1}{(1+r)^{j-1}} \int T \left( \sum_{i=1}^2 y_{i,j}(s) \right) d\Phi_j(s) = \sum_{j=T+1}^{T+T_R} \frac{1}{(1+r)^{j-1}} b + \sum_{j=1}^{T+T_R} \frac{1}{(1+r)^{j-1}} \left( \frac{G}{T+T_R} \right).$$

Because the objective function of the Ramsey problem is equivalent to a weighted sum of expected lifetime utilities of a newborn cohort, the advantage of this specification of government budget constraint is that there is not transition dynamics of the Ramsey problem.<sup>48</sup>

Figure 2.15 reports the inferred Pareto weights and the optimal response of income tax policy with respect to only economic changes with this specification of the government budget constraint. The results are quite close to the benchmark case, and the economic changes account for about 41% of the reduction in progressivity in the data.

<sup>48</sup>On the other hand, the disadvantage of this specification is that it rules out the possibility of a pay-as-you-go social security system.

## Alternative Objective Function of the Ramsey Problem

In the benchmark case, the objective function of the Ramsey problem is a weighted sum of lifetime utilities of each cohort at the stationary competitive equilibrium. An alternative way of measuring the welfare level at the stationary competitive equilibrium is to maximize the weighted sum of household flow utilities. With that measure, the objective function of the Ramsey problem is now

$$\left\{ \sum_{j=1}^T \int u(c_j(s), \{l_{i,j}(s), n_{i,j}(s)\}_{i=1}^2) W(A) d\Phi_j(s) \right. \\ \left. + \sum_{j=T+1}^{T_R} \int u^R(c_j(s)) W(A) d\Phi_j(s) \right\} + \gamma (T + T_R) \log \left( \frac{G}{T + T_R} \right).$$

Figure 2.16 reports the main quantitative results with this objective function of the Ramsey problem. The results are overall similar to the benchmark case, and economic changes now account for about 72% of reduction in progressivity we observe. This larger reduction in progressivity in response to economic changes is because the flow utilities of old households are no longer discounted by the  $\beta$ 's, and they are more sensitive to the amount of human capital accumulation. So as the return to human capital investment increases, the benefit of a less progressive income tax to encourage human capital accumulation is larger with this specification of the objective function than in the benchmark case, and the optimal tax policy reflects this difference with a lower progressivity in the end.

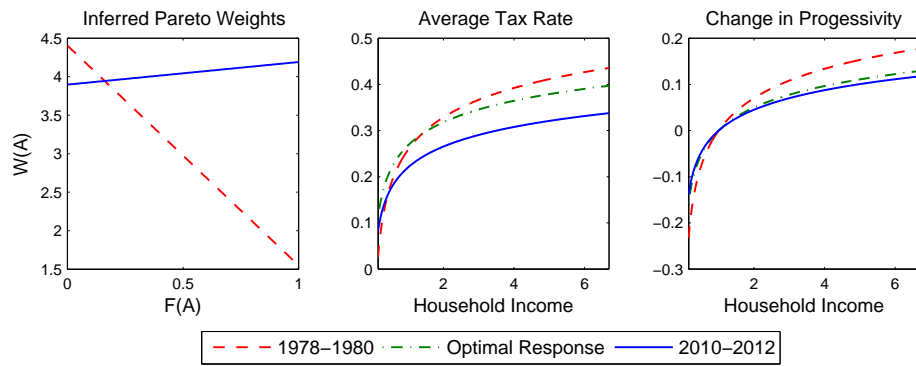


Figure 2.16: Quantitative Results with Alternative Ramsey Objective Function